TIME SERIES ANALYSIS

SMOOTHING BY STAGES

Lewis A. Maverick

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PREFACE

THIS book is concerned with trends or smoothing lines, and with cycles. In difficulty it is designed for the student or statistical clerk who has had but little training in statistics. He should have fitted straight line trends by least squares, and have calculated seasonal indexes and the normal line; he should be familiar with the use of the normal line as a base of reference in studying business cycles; he should have plotted time series, trends, and moving averages on quadrille paper and on semi-logarithmic paper; and he should be ready to form conclusions as to the relative advantages, as trends, of the straight line and sundry curves.

BUT in this Preface the work must be defended; consequently appeal is made here to more highly qualified readers.

THE process of time series analysis through "smoothing by stages" owes much to Ragnar Frisch, whose ideas underlie the whole. A minor difference from Frisch is in the emphasis upon moving averages rather than upon points of inflection (see the first reference, below, to Econometrica; in that article, the writer followed Frisch closely, even to the extent of relying upon points of inflection). Other important predecessors are Simon S. Kuznets and C. A. R. Wardwell.

THE method of smoothing by stages has been described or used by the writer in several publications: "Time Series, their Analysis by Successive Smoothings", Econometrica, I, 2, July 1933, pp. 238-246. "Cycles in Real Estate Activity", Journal of Land and Public Utility Economics, VIII, 2, May 1932, pp. 191-199. "Cycles in Real Estate Activity, Los Angeles County", ibid., IX, 1, Feb. 1933, pp. 52-56. "Real Estate Activity in California", ibid., X, 3, Aug. 1934, pp. 291-295. Economic and Social Statistics, University of California Press, 1941.

THE method was used and commended by Elizabeth Waterman Gilboy in "Time Series and the Derivation of Demand and Supply Curves: A Study of Coffee and Tea 1850-1930", Quarterly Journal of Economics, XLVIII, August 1934, pp. 667-685.

THE procedure has been improved and reduced to routine, so that now it seems appropriate to present a full report, with illustrations, in the hope that statisticians may find it useful.

A JUNIOR statistician who follows this method of analyzing time series can arrive at serviceable smoothing lines, of which that of the highest order will approximate the underlying or secular trend. The method gives him tools to accomplish a cycle analysis which formerly lay wholly beyond his powers. The smoothing lines and cycles give him material for a rich description of each time series, so that comparison between series can be made similarly extensive. This information enables him to make a good mechanical forecast; to be sure, a mechanical forecast is inadequate, and a really adequate forecast must always lie beyond the powers of anyone not master both of the technical processes of statistics and of the field studied; but the mechanical forecast is at least an excellent beginning, upon which someone more expert may make modifications to allow for expected forces and tendencies.

THE method of smoothing by stages, when employed by an accomplished statistician, saves no <u>time</u> in determining the trend line, for there is an unavoidable amount of detail in the process. But the highest order smoothing line probably furnishes a closer fitting trend than he can secure by a total algebraic process. And he may rest assured that for him too, as well as for the novice, the method gives new powers in the fields of cycle analysis, the comparison of time series, and forecasting.

MANY have undertaken cycle analysis, using other methods of segregating the cyclical movements from the non-recurrent components of the series. A leader among such statisticians is Simon S. Kuznets; extensive reference is made to one of his books in Chapter V herein, and to an article in this Preface. See also in Wesley C. Mitchell, <u>Business Cycles</u>, the chapter on Statistics. Analyses by these men and by others have shown the significance of this phase of time series study. It is hoped that statisticians may find that the method of smoothing by stages improves the tools for the study.

CONSIDER now a classification of types of trends, and place among the categories the smoothing lines secured under the method of "smoothing by stages". The chief reference will be to the searching theoretical article by Simon S. Kuznets, "On the Analysis of Time Series". (1)

TRENDS fall into two broad classes: empirical, and mathematically fitted by some total process. An empirical trend grows out of the data in its own vicinity in time, by an inductive process. It fits close to the plotted points representing the data. It has no preconceived form, and when it has been located, it usually defies description by a mathematical equation. Examples of empirical trends are moving averages and lines drawn free-hand. As will be seen, the smoothing lines secured through the method of "smoothing by stages" combine the properties of these two sub-classes. Most critics have granted that the moving average is satisfactorily objective (the subjective element lying principally in the choice of the length or period of the average), but some contend that the free-hand trend has been so subjective as to call for complete rejection. In the process of smoothing by stages, there is some departure from the moving average in the direction of a free-hand curve, but the process is protected by several objective criteria.

KUZNETS, in the article just referred to, "On the Analysis of Time Series", questions whether an empirical trend can contain enough internal evidence of the persistence of form through successive periods, to warrant a forecast. In response, it may be pointed out that the moving average shows the actual local central tendency of the variable through each cycle,with a faithfulness to current conditions that cannot be approached by a trend line fitted by a total mathematical procedure. Consequently, any persistence of form in the moving average furnishes a much better basis for a forecast than does a similar apparent persistence of form in a total mathematical curve. To be sure, persistence of form can be shown more reliably by another mathematical procedure: if the period be broken into parts and a trend fitted separately to each short part, the series of trends so secured will give a satisfactory basis to judge persistence of trend form; but this procedure is quite different from fitting one trend to the totality of the data.

THE second general class of trends comprises those that are mathematically fitted - usually by a total process; some description of this second type of trend has already been offered, for contrast, in discussing the first type. One and sometimes two decisions involving subjective judgment are

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(1) Journal of the American Statistical Assn. *XXIII, 1928, pp. 398-410.*

required: (a) what type of curve to fit; and sometimes, (b) a critical date or other parameter, such as the date of the point of inflection of a logistic curve. Trends that have been mathematically fitted are sufficiently objective to satisfy the requirements of economic statistics.

THE commoner types of curves employed in the mathematical process are: (1) the straight line, (2) the parabola or second degree polynomial, (3) the cubic or third degree polynomial, sometimes called a third degree parabola, (4) the simple exponential curve, which appears as a straight line on semi-logarithmic paper, and (5) the logistic and Gompertz curves, which are characterized by an S-shape. Other forms have been considered: higher degree polynomials are not practical; the arc-tangent might be added in class 5; and recurring trigonometric functions like the sine and cosine might form a sixth class; they have been of interest to statisticians who think of the long tidal movements.

ONE may distinguish - though it is of theoretical interest only - between two types of mathematically fitted curves: on the one hand, curves that really essay an explanation of the changes in the variable under study --- with parameters that correspond to real phenomena. Such curves have not been discovered for economic time series. On the other hand, there are curves which serve as smoothing devices. These differ in the amount of explanation they seem to offer of the phenomena under study, and in the "reasonableness" of their shape. Kuznets, in the book to be examined in Chapter V, offers the S-shaped logistic curve as the "proper" trend form for industrial growth, etc., although he is unable to give physical meaning to the parameters in the equation.

BUT no one has yet come forward with a general type of curve to fit price series. In a period of stable money (or if correction were made for the changing general value of money), a straight line or a logistic might fit. This problem will be referred to again, below.

AN important theoretical issue between empirical trends and those fitted by a total mathematical process, is with respect to the assumption of homogeneity of the forces affecting the value of the variable. "If . . we have forecasting done from a single line of trend, from a description that is . . . . historically limited, the assumption is that the forces that have been determining such movement in the past will continue to do so in the future - will repeat themselves. The basis of expectation here is not at all the statistical analysis, but information from a different source, which enables the forecaster to assert that the period for which the trend line was fitted was homogeneous, that is, under a preponderant influence of one and the same known set of forces, which is expected to repeat its influence in the future". (1)

TO illustrate how radical and unrealistic is the assumption of underlying homogeneity of the affecting forces, even through the observed period - - without extrapolating into the future - - - let us consider several time series. Suppose one were studying the method of lighting in American homes since 1800 (or the financial outlay upon that lighting, or the total candle power). Heterogeneity is striking, for the homes have been lighted by oil lamps, candles, kerosene, gas, and several types of electric lamps.

IF a price series were under examination, not only would there be involved problems of supply (discoveries, exhaustion of resources, etc.) and demand in the single industry (in which homogeneity might not be impossible), but also in the supplying industries, the rival industries, and the industries which use the product of this one as their raw material. An invention, or

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(1) Kuznets, "On the Analysis of Time Series", p. 400.

a change in import or excise taxes, in any of these fields, would change the underlying forces. And, most fickle of all, the changing purchasing power of money makes for heterogeneity over time, in any price series.

IT would be unwise to fit a smooth mathematical curve to the number of votes cast in American presidential elections. There have been changes from property qualification to universal white manhood suffrage, to the freeing of the slaves, and to woman suffrage; territorial growth from thirteen states to forty-eight; the Civil War as an affecting episode; changes in the flow of immigration and of the westward movement of population; and the subjects voted upon have also changed, as for example in the matter of the direct election of senators.

FOR the business of the Port of San Francisco, homogeneity cannot be predicated, for the record runs through Spanish, Mexican, and American sovereignty, the gold strike of 1848, the Civil War, the completion of the transcontinental railroad in 1869, and of other lines, the Spanish-American War and the resulting development of far eastern trade, the earthquake and fire of 1906, the first World War, the opening of the Panama Canal in 1915, and the Second World War - with the emergence of air traffic.

QUITE naturally, the assumption of homogeneity of the affecting forces often strikes the operator himself as untenable; see, in Chapter V, how Kuznets has broken into two fragments the trend that he fitted to the series on Erie Canal freights. And, even when the fit of a mathematical curve is not so bad as to demand such fragmentation, it may, nevertheless, be worse than the fit of a well-adapted empirical curve of the same gentleness (long radius) of curvature.

EVEN in the case of fitting an empirical trend or smoothing line, it may sometimes prove advisable to regard some of the data as so completely different from the rest that the empirical line should be made discontinuous. An excellent recent example of such treatment is to be found in Norman J. Silberling, Dynamics of Business, (1) page 154. Silberling, in dealing with price series, regards the inflationary episodes of wartime as belonging in another "statistical universe" from prices during peacetime. Consequently, he discontinues his smoothing line through those inflationary periods.

THE present writer faced another problem in studying the various business series of San Francisco as they were affected by the earthquake and fire of 1906. I had plotted monthly data. In the first smoothing line, SL A, I allowed a saltatory displacement to stand, at the time of the catastrophe. In the second smoothing line, SL B, which cut through the short business cycle, I again permitted a saltatory displacement, though along a somewhat sloped line, rather than abruptly vertical. As for the third smoothing line, SL M, which cut through the major cycle, I felt it advisable to draw the curve in a continuous manner rather than to permit again a saltatory displacement. But I recognized the subjective nature of the decision; possibly another saltatory decline should have been admitted, and the continuous line reserved for the next stage of smoothing, designed to remove the long wave.

AN important publication in the field of the present book is Macaulay: The Smoothing of Time Series. Macaulay provides a number of weighted formulas for moving averages, designed for series in which the length or period of the cycle is reasonably uniform. Some of Macaulay's formulas have the advantage of giving greater weight to the middle values and less to

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(1) McGraw-Hill, 1943.

the ends. This brings the moving average to the full desirable departure in the convex direction (departure from the less satisfactory position which an unweighted average would occupy), in periods when there is marked curvature in the moving average. But that same purpose is accomplished herein by the reiteration of the moving average (see correction for curvature, Chapter II), with an advantage over Macaulay's formulas, of adaptation to changing lengths of the cycles.

C. A. R. WARDWELL devised the "moving cyclical average", which makes possible an objective check upon the smoothing process for series with changing cycle lengths. His moving cyclical average is here accepted as the principal objective check in the smoothing process. Some ingenious statistician may find a way to combine Wardwell's contribution of the variable length moving average with Macaulay's heavy weighting of the central values, and so make unnecessary a separate calculation to correct for curvature. But that separate calculation is not laborious, and gives excellent results.

THE method of smoothing by stages is systematic; each stage of smoothing is similar to the next. The only difference among the stages is in one detail. In smoothing out a daily, weekly, or annual cycle, a simple moving average with fixed and uniform length is employed, because each successive cycle has the same length. But in smoothing out the short business cycle, the major cycle, or the "long wave" - - and also in smoothing out the monthly cycle (the months consisting of 31 days, 28, 31, 30, etc.) - - Wardwell's moving cyclical average of changing length is used. But this minor adaptation of method does not impair the truly unified and systematic nature of the procedure.

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> . . . "the analysis became necessary since recurrent changes had to be separated from the non-recurrent ones, and . . . the recurrences of different amplitude and duration had to be distinguished from one another." Kuznets (1)

CHAPTER I.

INTRODUCTION

THE method of analyzing time series which we shall call "smoothing by stages", is primarily graphical. One arranges the data in two forms, as numerical values in a table, and as a time polygon upon a chart.

HE draws a "first smoothing line", which cuts through and "removes" the cycle of shortest period (the word "removes" means that the smoothing line is made completely free from that short-period cycle). Then through the fluctuations remaining in the first smoothing line, he draws a second smoothing line, which removes the cycle or fluctuation of next shortest period; etc., until in the final smoothing line there remain no recurrent movements. If the values of the time series have been given at monthly or quarterly intervals, his first task is to remove the seasonal fluctuation - which in its full historical record may be called the annual cycle - by the application of Smoothing Line A (so-called because it removes the annual cycle; abbreviated as SL A). Then SL A, in its turn, is smoothed by a second order smoothing line, SL B, which cuts through and thereby "eliminates" the short business cycle, and which derives its name from the initial letter of that cycle. But if the data are in annual form (instead of monthly or quarterly), the first task is to cut through the fluctuations of the short business cycle by drawing SL B. In this book, except for the brief illustration in this Introduction, all the examples have annual data, so that SL B is the first smoothing line obtained. SL B, in turn, is smoothed by the application of SL M, which eliminates the major cycle, a fluctuation ten to thirty years in length. The highest order smoothing line obtained (usually it is SL M) serves as an approximation to the underlying "secular" trend, the trend through the centuries.

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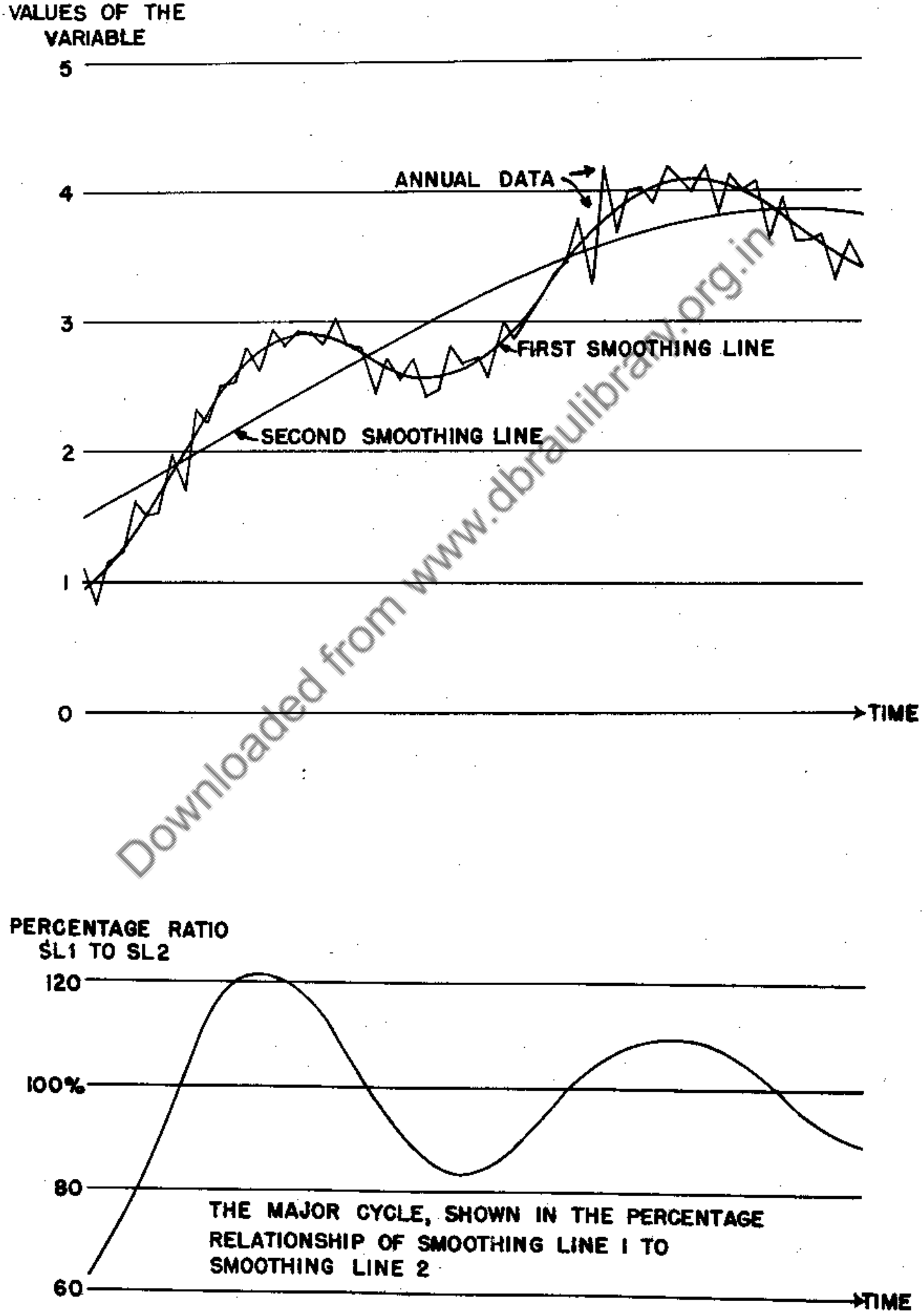
(1) *Simon S. Kuznets, "On the Analysis of Time Series"*, Journal of the American Statistical Association, *XXIII, 1928, page 399.*

SKETCH a, SMOOTHING LINES AND CYCLE



THE MAJOR CYCLE, SHOWN IN THE PERCENTAGE RELATIONSHIP OF SMOOTHING LINE 1 TO SMOOTHING LINE 2

IN the relationship between any pair of successive smoothing lines, is found the history of one order of cyclical movement. The relationship between the original monthly or quarterly data and SL A gives the cyclical and irregular movements of the shortest order, termed the seasonal movement or the annual cycle. The relationship of the first order smoothing line, SL A, to the second order line, SL B, gives the history of the cyclical-irregular movement commonly called the short business cycle. In case the original figures are annual, instead of monthly or quarterly, the short business cycle is revealed in the relationship between the annual data and SL B. The relationship of SL B to SL M gives the major cycle, as in Sketch a; it is this movement that is marked by the great booms and deep depressions. In series longer than about 80 years, there may be found another such cyclical relationship, the "long waves" that have been studied by Kondratieff and others. In the study of some one order of fluctuation, the full history of the ratio of the lower order line to the higher is of interest. But it is of equal, and possibly greater importance, that from this extended history, by inductive steps, there may be derived a typical pattern, somewhat uniform and constant; and that certain standard measures of that typical cycle may be calculated. When the standard measures have been calculated for each order of fluctuation separately, excellent material becomes available for a forecast - for a more thorough forecast than has yet lain within the power of statisticians. This forecast makes use of the standard measures of the several orders of fluctuation; it is realistic, extensive, and helpful; It is hoped that it may be generally accepted as a decided improvement on the customary forward extension of the normal line.

TABLE A and Chart 1 will show that there already exist ways of segregating or isolating the annual or seasonal fluctuation from the other movements in a time series. This table and chart do not illustrate technically the method of "smoothing by stages" but merely serve as an introduction to it.

IN Table A and on Chart 1, a twelve month moving average is fitted; in the case of this particular series, the curve connecting the average points is found to be smooth; consequently it will serve, without modification, as an acceptable approximation to SL A, and as a good base of reference for the study of the annual cycle. Under the method of smoothing by stages, the moving average curve might be more carefully smoothed; but, practically speaking, that further refinement is not often necessary unless the twelve month moving average curve is quite irregular. In Chapter II, the recommendation will be made that monthly data be consolidated into quarterly figures before undertaking the smoothing process. That consolidation makes it even more unlikely that irregularities will disturb the smooth flow of the four quarter or "annual" moving average.

Table A. SAN FRANCISCO REAL ESTATE ACTIVITY, 1920 TO 1929

The annual cycle, as shown in the ratio of actual value to moving average.

Source: San Francisco Real Estate Circular, Thomas Magee and Sons.

Month	Number of Deeds	Moving Average, 12 months length		Percentage Ratio, Actual to Moving Average	Trend (fitted by least squares)
		not recent-ered	recent-ered		
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1920 Jan	838				1034
Feb	759				1035
Mar	969				1036
Apr	771				1037
May	812				1038
June	676				1039
July	706	767	764	92	1040
Aug	645	761	758	85	1040
Sept	766	756	754	101	1041
Oct	791	753	758	104	1042
Nov	768	763	765	100	1043
Dec	702	767	772	91	1044
1921 Jan	764	778	784	97	1045
Feb	702	789	791	89	1046
Mar	928	793	791	117	1046
Apr	873	789	790	110	1047
May	883	790	796	110	1048
June	803	803	807	100	1049

Month	Deeds	Moving Average		Ratio	Trend
		not recent-ered	recent-ered		
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1921 July	848	811	822	103	1050
Aug	685	834	844	81	1051
Sept	720	854	866	83	1052
Oct	901	878	891	101	1052
Nov	824	904	910	91	1053
Dec	805	915	921	87	1054
1922 Jan	1030	927	932	111	1055
Feb	945	937	953	99	1056
Mar	1215	969	975	129	1057
Apr	1195	981	994	120	1057
May	1006	1008	1021	99	1058
June	952	1034	1038	92	1059
July	947	1043	1051	90	1060
Aug	971	1059	1070	91	1061
Sept	986	1082	1092	90	1062
Oct	1218	1101	1115	109	1062
Nov	1135	1129	1144	99	1063
Dec	919	1160	1176	87	1064

Table A (continued) San Francisco Real Estate Activity

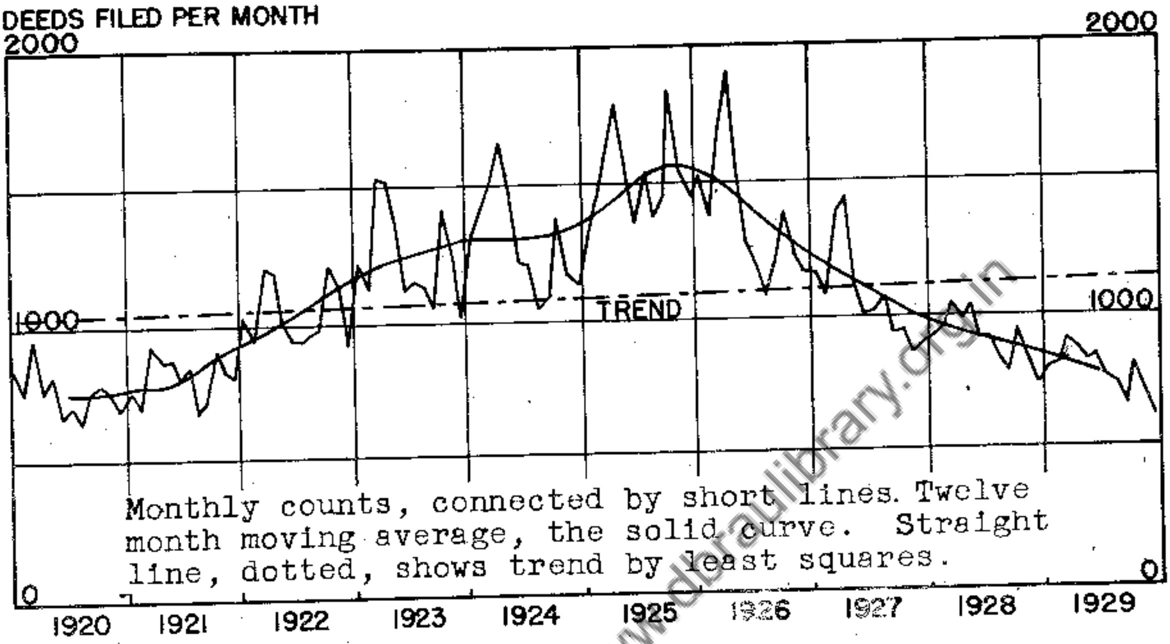
Month	Deeds	Moving Average		Ratio	Trend	Month	Deeds	Moving Average		Ratio	Trend
		not recent-ered	recent-ered					not recent-ered	recent-ered		
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T	X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1923 Jan	1217	1191	1192	102	1065	1924 Oct	1379	1313	1318	105	1083
Feb	1121	1192	1199	94	1066	Nov	1182	1324	1326	89	1084
Mar	1544	1206	1208	127	1067	Dec	1141	1328	1335	85	1085
Apr	1527	1211	1219	125	1068	1925 Jan	1301	1342	1356	95	1086
May	1388	1227	1232	112	1068	Feb	1453	1371	1381	105	1086
June	1119	1238	1242	90	1069	Mar	1608	1391	1410	113	1087
July	1161	1245	1249	93	1070	Apr	1801	1429	1454	124	1088
Aug	1135	1253	1266	89	1071	May	1529	1478	1484	103	1089
Sept	1047	1278	1278	82	1072	June	1366	1490	1507	91	1090
Oct	1412	1277	1282	110	1073	July	1552	1524	1533	101	1091
Nov	1251	1288	1292	97	1074	Aug.	1383	1542	1539	89	1092
Dec	1018	1295	1298	78	1074	Sept	1461	1536	1540	95	1092
1924 Jan	1321	1302	1304	101	1075	Oct	1841	1543	1548	119	1093
Feb	1416	1306	1298	111	1076	Nov	1541	1552	1552	99	1094
Mar	1521	1290	1296	110	1077	Dec	1446	1552	1550	93	1095
Apr	1666	1302	1301	128	1078	1926 Jan	1529	1547	1547	98	1096
May	1468	1300	1297	113	1079	Feb	1381	1547	1530	90	1097
June	1210	1294	1299	93	1080	Mar	1690	1513	1500	113	1098
July	1207	1304	1303	93	1080	Apr	1908	1488	1477	128	1098
Aug	1046	1302	1304	80	1081	May	1526	1466	1448	105	1099
Sept	1093	1306	1310	83	1082	June	1275	1429	1416	90	1100

Table A. (concluded) San Francisco Real Estate Activity

Month	Deeds	Moving Average		Ratio	Trend	Month	Deeds	Moving Average		Ratio	Trend
		not recentered	recentered					not recentered	recentered		
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T	X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1926 July	1172	1402	1382	85	1101	1928 Apr	991	927	927	106	1119
Aug	1083	1362	1355	80	1102	May	1050	927	927	113	1120
Sept	1204	1348	1336	90	1103	June	908	927	916	98	1121
Oct	1388	1323	1306	106	1104	July	913	915	910	100	1122
Nov	1217	1290	1275	95	1104	Aug	843	905	900	94	1122
Dec	1170	1260	1243	94	1105	Sept	790	894	894	88	1123
1927 Jan	1164	1226	1215	95	1106	Oct	952	882	888	100	1124
Feb	1084	1204	1198	90	1107	Nov	858	872	877	97	1125
Mar	1394	1192	1192	117	1108	Dec	752	854	851	88	1126
Apr	1448	1191	1180	113	1109	1929 Jan	799	848	842	95	1127
May	1117	1169	1150	97	1110	Feb	819	837	833	98	1128
June	1009	1132	1117	90	1110	Mar	905	829	824	109	1128
July	1023	1102	1091	94	1111	Apr	880	820	814	108	1129
Aug	1074	1080	1070	100	1112	May	827	808	802	103	1130
Sept	936	1059	1054	89	1113	June	841	795	790	107	1131
Oct	947	1049	1034	92	1114	July	776	785			1132
Nov	864	1020	1001	86	1115	Aug	751				1133
Dec	900	982	979	92	1116	Sept	675				1133
1928 Jan	918	976	968	94	1116	Oct	812				1134
Feb	955	959	950	100	1117	Nov	707				1135
Mar	1050	940	934	112	1118	Dec	624				1136

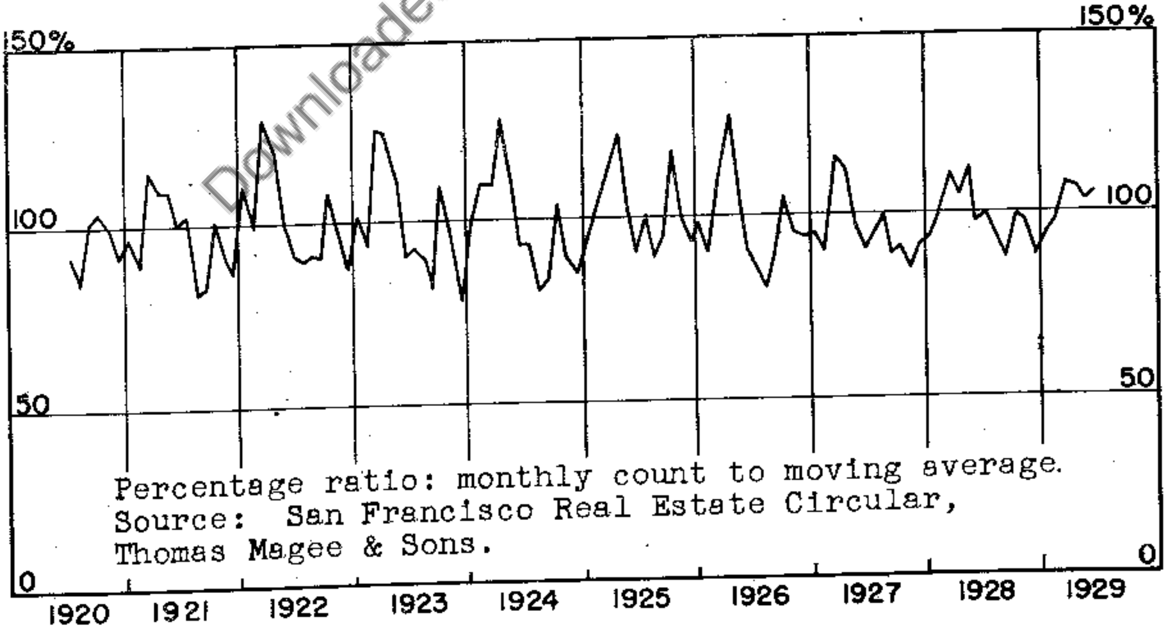
CHART I.
SAN FRANCISCO REAL ESTATE ACTIVITY, 1920 TO 1929

(a) APPLICATION OF MOVING AVERAGE AND TREND



Monthly counts, connected by short lines. Twelve month moving average, the solid curve. Straight line, dotted, shows trend by least squares.

(b) THE ANNUAL CYCLE



Percentage ratio: monthly count to moving average.
Source: San Francisco Real Estate Circular, Thomas Magee & Sons.

PLAN OF PROCEDURE, SUMMARIZED.

THE study of time series by the method here presented, "smoothing by stages", falls into three phases;

1. Locate the set of smoothing lines (Tables A, B, and C; Charts 1a, 2, and 3).
2. Plot the several orders of cycles and determine their standard measures (Tables A, D, and E; Charts 1b, 4, and 5). This phase concludes the analysis proper.
3. Use the smoothing lines and standard measures obtained in phases 1 and 2 - in correlation and in forecasting (Chart 6).

CHAPTER II is devoted to the first phase, the location of the smoothing lines; Chapter III to the second phase, the study of the cycles; Chapter IV briefly treats the third phase, the use of the smoothing lines and the standard measures of the cycles; and Chapter V applies the first two phases to a group of seven time series.

A SIMPLIFIED FREEHAND PROCEDURE, WITH NO OBJECTIVE CHECKS.

FOR quick exposition, a simplified sketch of the process will now be given, a purely graphical procedure, without the objective check that is afforded by moving averages. Assume forty or fifty years[1] record of quarterly data.

PLOT the quarterly data on a time chart similar to Chart 1a, either on quadrille paper (charting paper with ordinary rectangular ruling)or semi-log paper. Connect the plotted points by straight lines, to form a time polygon.

THE first stage of smoothing is from the time polygon to Smoothing Line A, a line to be freed from all seasonal movements, both of a recurring form (the standard seasonal pattern), and episodes or short-length non-recurring movements; but this SL A still to exhibit in full the movements of the short business cycle, the longer cycles, and the trend. The following criteria should be observed in sketching the smoothing line:

(a) It should be made a smooth-flowing curve without sharp angles or short-radius turns.

(b) It should intercept from the time polygon connecting the plotted quarterly values, a series of plus and minus areas which show an approximate running balance or equality, above and below.

(c) It should seldom leave to one side (whether above or below) more than two consecutive quarterly points, and probably never more than three.

HAVING drawn Smoothing Line A, one undertakes the second stage of smoothing; he works from that line as his base of reference, to locate Smoothing Line B, a line designed to cut through and therefore to be freed from the short business cycle, yet to retain in full the movements of the major cycle and the trend. Through the short cycles exhibited in SL A, draw SL B, which by comparison will be a simpler or flatter curve, containing no residual movements of the short cycle (and of course no seasonal movements). Again observe the principle of a running equality of areas intercepted above and below. Try to keep the length of the intercepted cycles reasonably uniform (it is suggested that they may prove to be from two to six years in length).

IN like manner, proceed to the third stage of smoothing, drawing SL M to eliminate the major cycle, the 10 to 30 year fluctuation exhibited in SL B. It may be possible to proceed to a fourth stage of smoothing, if the record of data is sufficiently long.

THE values at assigned dates, of each of the successive smoothing lines, may be read from the charts (as from Charts 2 and 3), and transcribed to working tables (as Table Cb).

THE full historical record of the relationship between the monthly or quarterly data and SL A, constitutes the annual cycle or the seasonal movement; this relationship will appear as a series of values of the quarterly or monthly ratio. (1) This ratio $\frac{\text{actual}}{\text{SL A}}$ is ordinarily multiplied by 100 to convert it to a percentage ratio. Its successive values may be listed in a table (as Table A, Column 5), and depicted graphically (as on Chart 1b). From the record of this ratio, one can calculate the typical seasonal pattern.

SIMILARLY, the relationship between SL A and SL B, as indicated in the history of the ratio $100\,\frac{\text{SL A}}{\text{SL B}}$ gives the short business cycle. The values of this ratio may be entered in a table (as in Table Db, Column 2, and in Table Ea, Column 2), and may be shown graphically (as on Chart 4). The problem of calculating the standard or typical pattern of the short business cycle is more difficult than for the annual cycle (from which one calculates the seasonal pattern), for here there is a variable length or period.. However, a reasonably satisfactory standard pattern can be calculated, as will be discussed in Chapter III below, and as is shown on Chart 5.

THE major cycle is found in the relationship between SL B and SL M. This may be studied in the same fashion as has been suggested for the short cycle.

THE subjects of correlation and forecasting will be postponed to Chapter IV.

THIS concludes the preliminary exposition, in which the smoothing has been freehand, without benefit of the objective check of moving averages. For a good many applications, this freehand method is sufficiently accurate; its major defect is that it is not objective -- that two statisticians would not get precisely the same smoothing lines, and consequently the reader could not wholly trust the results. In order to make the process objective, and therefore acceptable in accordance with good statistical practice, it is necessary to check the graphical procedure; for this check, moving averages have been found useful.

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(1) The seasonal movements and the longer cycles may be studied on the basis of differences *instead of ratios; but for simplicity the exposition will be confined to ratios.*

CHAPTER II.

THE SMOOTHING LINES

GIVEN a time series made up of data at regular intervals, (1) which is to be analyzed by the method of "smoothing by stages": the first task in the analysis, the one which is to occupy this chapter, is to locate the successive smoothing lines, each of which will in turn cut through or eliminate an order of fluctuation. The first such line will eliminate the first or shortest order of fluctuation from the time polygon of the original data, the second will eliminate the shortest fluctuation still discernible in the first smoothing line (this is the second order of fluctuation in the time polygon of the original data), the third will eliminate another fluctuation from the second smoothing line, etc.

Section 1. LOCATING SMOOTHING LINE A.

ASSUME that quarterly data are supplied. The first line to be located will then be Smoothing Line A; it will cut through and thereby "eliminate" the seasonal or annual cycle. (It is not recommended that monthly data be plotted on the chart; the elaboration is great, and no value derives from it. For the purpose of the present chapter, which is merely to locate the smoothing lines, it would be better to consolidate monthly data into quarterly form before plotting and smoothing. Subsequently, when the several smoothing lines have been located, and the attention turns to the study of the annual cycle, the operator may choose to make that study on a monthly basis. He has only to read from the chart the values of SL A at monthly instead of quarterly intervals, and to compare those readings with the original monthly data.)

PLOT the quarterly data either on quadrille paper or on semi-log paper.(2) Connect the quarterly points by straight lines, forming a time polygon.

BEGIN Table B (see also Table C); 24 columns will be indicated, as that one table serves all three stages of smoothing. In column 1, enter the dates of the quarterly figures; and in column 2, their values. Calculate a four quarter moving average. The sole purpose of this moving average will be graphical, to serve as a guide to the desired SL A. Note that the arithmetic type of moving average may be used in this stage of the analysis, even though the chart be on semi-log paper. See statement below in this chapter (Section 2), on the use of a moving geometric mean in the later stages of smoothing, if the chart is on semi-log paper.

ENTER the moving average values in column 3, at levels to show that they fall between the dates of column 1; plot them on the same chart with the time polygon of the data. It is not necessary to recenter these averages, as would be the case if they were to be used directly in the calculation of seasonal ratios. Since they are to be used only graphically, as aids in the locating of SL A, it is actually an advantage to have them fall on the chart halfway between the dates corresponding to columns 1 and 2. The

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(1) Should the data be at irregular intervals, the method of smoothing by stages is still applicable, as in the early years of the Erie Canal freight series in Chapter V. but the first stage of smoothing may then need to be principally freehand.

(2) The decision as to the type of ruling to be favored will not be discussed. See the seven series in Chapter V.

line connecting these moving average points will be found to be almost entirely freed from seasonal movement, but still to contain the short business cycle, the major cycle, and the trend. This line will give a close and dependable guide to the desired SL A (see Chart 1). However, there may still be found some residual seasonal irregularities; hence it may be necessary for SL A to depart from the moving average (ma) points slightly, in order that SL A may be completely freed from even the irregular movements of approximately the length of the more regular seasonal cycle.

DRAW SL A, a flowing curve, following the moving average points fairly closely, but pursuing an intermediate course between any irregular high and low values. The principle should be observed of a running equality of areas above and below this smoothing line. Smoothing Line A should follow the quarterly moving average points so closely that, save at peaks and troughs (peaks and troughs require special attention; see discussion of curvature, below, in this chapter), one will seldom find more than two consecutive moving average points lying on the same side (either above or below). Six or eight successive points may occasionally be allowed to lie on one side, if the data and averages occur at monthly intervals.

SL A is designed to remove completely both the regular seasonal pattern and any short length irregular movements, but it should not do more than this, for it is not desired at this stage to smooth out any portion of the short business cycle, nor any portion of the major cycle. The reason for this caution will become clear in the study of cycles, Chapter III.

IN carrying SL A nearer to either end of the series than six months, employ a dotted line, which will indicate the tentative or provisional character of the line near the end of the distribution (see discussion of moving averages, below).

READ the values of SL A from the chart at quarterly intervals, at the same dates as are entered in columns 1 and 2, and enter these values in column 4. The reason for reading these values at the same dates as those in column 2, is that the seasonal movement (the annual cycle) will be studied by examining the ratios of the data to SL A, and for that purpose it is necessary to have the two sets of figures at simultaneous dates.

(IN the numerical examples which follow, the data will be supplied in annual form; consequently the first smoothing line to be secured will be SL B).

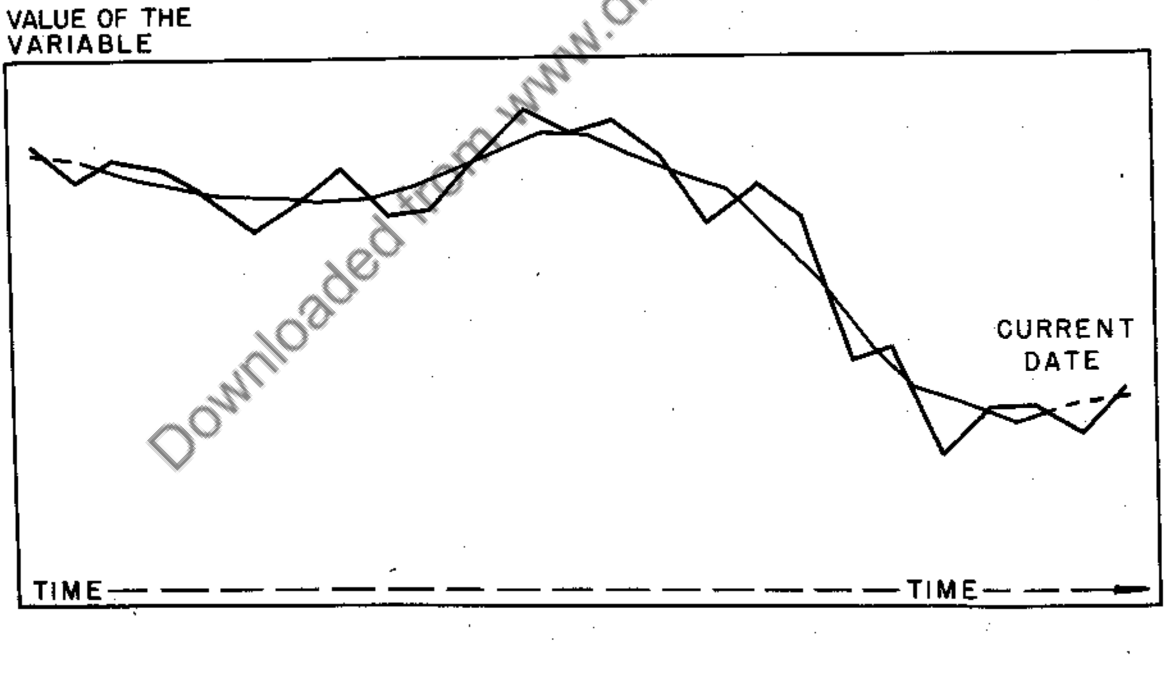
Section 2. DEVICES TO AID THE SMOOTHING PROCESS.

MOVING AVERAGES.

THE method of smoothing here employed is in part based upon a moving average procedure, similar to that shown in Table A and on Chart 1. We shall be concerned with two types of moving averages, those of uniform length, and those of varying length. In smoothing quarterly data to eliminate the annual cycle, the length or period of the cycle is a constant, four quarters, and therefore the moving average to eliminate the annual cycle is taken of that same constant length. The four quarter moving average removes the seasonal or annual cycle from the curve of the data, without disturbing or removing the short business cycle, the major cycle, or the trend; in other words, the short business cycle exhibited by the curve of the moving average points is precisely the same as the short business cycle exhibited by the original data - and so are the major cycle and the trend identical.

AS is commonly known, moving averages cannot be brought abreast of the current date. Smoothing lines retain this shortcoming of the moving averages upon which they are built -- that they cannot with confidence be brought to the present date. Consequently, when it is found necessary to estimate the current value (the current "ordinate") of one of the smoothing lines -- and the caution applies still more when effort is made to forecast a future value -- one needs to treat that current or future value as approximate and tentative. The precise value of Smoothing Line A for February, 1954, will not be reasonably assured until that date has slipped six months into the past; because the four-quarter moving average, and Smoothing Line A, are built upon data a full year in length,extending six months in both directions in time. For SL B, about two years must pass before the ordinate may be considered well established; and for SL M, ten or twelve years.

SKETCH b, TO SHOW DOTTED ENDS OF MOVING AVERAGE LINE

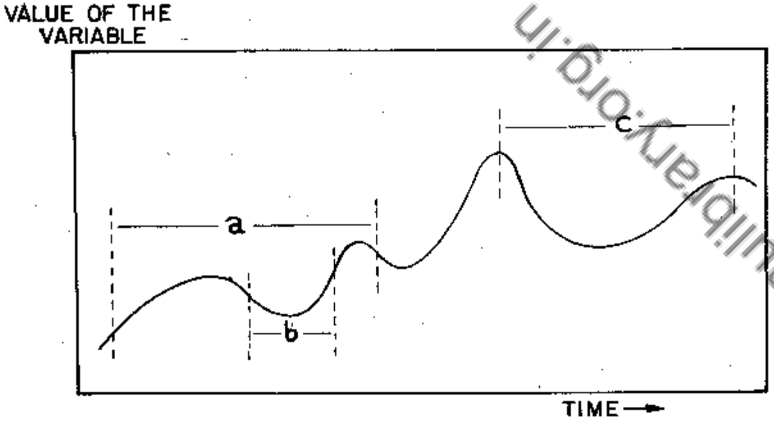


THE MOVING CYCLICAL AVERAGE.

THE short business cycle and the major cycle are characterized by variable length; sometimes the short cycle is scarcely more than a year, but at other times it may prove to be eight or nine years long: sometimes the major cycle is but twelve years long, while at other times as many as thirty years may pass between major depressions. A modified form of the moving average, the "moving cyclical average", permits varying the length or period of the average, from cycle to cycle, so that at each application precisely one cycle will be averaged, and so smoothed out. This type of moving average was devised by C. A. R. Wardwell of Northwestern University The important feature of the moving cyclical average is that the length of each successive average is precisely one cycle.

IN working from a tabulated and charted Smoothing Line A, in the attempt to locate Smoothing Line B, it would be a mistake to use as the length of the average, a period of time that did not correspond precisely to one cycle as exhibited in the relation of SL A to SL B; that error would result in a calculated value (the ordinate) of the average,which would in general fail to attain the intermediate position or average value which should characterize SL B. If for example (a, in Sketch c), two peak periods in SL A should be included in the period taken for the moving average, and only one period of depression or inactivity, the average based on that badly chosen interval would be improperly weighted, and its value would be found too high to afford a useful guide to the desired Smoothing Line B.

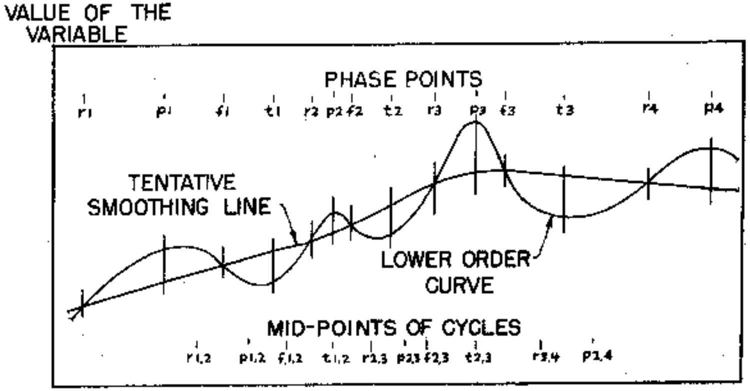
SKETCH c, CORRECT AND INCORRECT LENGTHS FOR MOVING AVERAGE



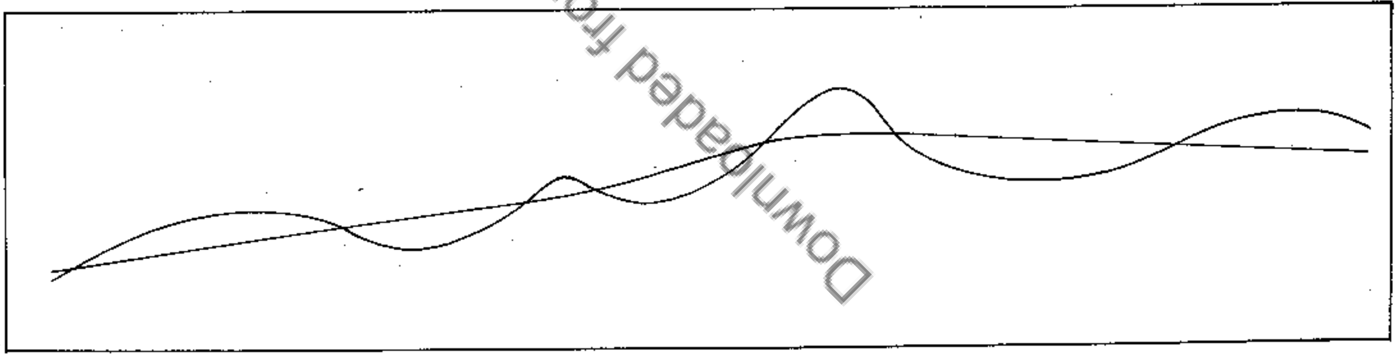
Sketch c, Correct and Incorrect Lengths for Moving Average.

(a) first error: including two peaks and only one trough; consequently getting too high a value of the average to serve as a sound guide to the smoothing line.
(b) second error: including only a half-cycle; in the case shown, the resulting average would be too low.
(c) correct time length: the average will be truly representative of the series in the indicated interval of time.

SKETCH d, TENTATIVE SMOOTHING LINE AND PHASE POINTS



SKETCH d2, SAME AS SKETCH d, WITH EXTENDED TIME SCALE



TENTATIVE SMOOTHING LINE AND PHASE POINTS

IT is advisable to make use of a moving average composed of a succession of averages each based upon a period precisely one cycle in length, as observed in those particular years. (1) In order to accomplish this result, use may be made of two related devices: a tentative smoothing line and a number of phase points.

IN each cycle, there are four points easy to identify: the peak, the trough, and the two points approximately half-way between those two extremes; these four may be called phase points. The quarter cycles between successive phase points may be called the four phases of the cycle. These terms have been borrowed from the study of wave movements in physics.

START with a tabulated and charted "lower order" curve, to which a smoothing line is to be fitted. Through the fluctuations in the lower order curve, draw a tentative smoothing line (T SL) to cut through the short-period fluctuations; employ a wholly graphical procedure resting upon a running equality of areas intercepted, and gentle flowing curvature, as described in the Introduction (Chapter I). To locate the phase points, inspect the manner in which the lower order curve fluctuates about the tentative smoothing line as a central tendency. Locate those points in the lower order curve at which the ordinate above T SL is a maximum; call these points peaks, and give them the designating letter p (the successive peaks may be numbered p_1, p_2, p_3, etc.). Locate the points farthest below the T SL; and call these points troughs, t. Call the intersections of the lower order curve with the T SL, r or f, according as the lower order curve is rising from a previous trough, or falling from a previous peak, at the intersection.

WHEN you locate a phase point, let your attention be fixed on <u>the abscissa</u>, not the ordinate -- on the date, <u>the instant of time</u>, rather than on the value of the variable. Consequently, although it may be convenient first to mark the phase points directly on the curve, it will be well to show them also by short vertical marks along a horizontal line.

THE elapsed time between two successive phase points gives a phase (roughly a quarter) of a cycle: r_1p_1, p_1f_1, f_1t_1, t_1r_2, r_2p_2, etc. The elapsed time between any particular phase point and the similarly named (homologous) phase point in the next cycle (from one peak, as p_2, to the next, p_3; from one trough, t_5, to the next, t_6; from one r to the next; or from one f to the next) gives an exact cycle length. It will be observed that these "cycles", measured from four different starting points in each cycle, overlap like shingles or like moving averages; but they differ from the usual moving average in that the number of months in any cycle is determined by the dates of the phase points at its beginning and its end; the number of months comprised in one cycle is likely to be different from the number of months in the adjoining and overlapping cycles. (See Table C and Charts 2 and 3).

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(1) If the reader wishes to pursue this subject further, he is respectfully referred to the writer's article in Econometrica, *July 1933, and to the book by C. A. R. Wardwell,* An Investigation of Economic Data for Major Cycles, *Northwestern University.*

Section 3. LOCATING SMOOTHING LINE B.

(Tables B and C, and Charts 2 and 3).

INSPECT the chart on which SL A has been located. If there are so many construction lines as to cause confusion in the further construction, transcribe SL A to a new and clean chart. Often in this transcribing, it will be found helpful to condense the time scale, in order to make the business cycle stand out; there is no longer need for close detail, for there is in SL A no residue of the seasonal variation. (See the contrast between the condensed time scale on Sketch d, and the extended scale on Sketch d2.)

TENTATIVE SL B should be drawn through the fluctuations in SL A, in freehand fashion: it should cut through and eliminate the short business cycle. Take care to secure a running equality of areas intercepted above and below, and make Tentative SL B follow a gentle curvature, avoiding short-radius turns and sharp angles. Tentative SL B should exhibit in its own movements the trend and the major business cycle, for it represents an attempt to remove the short business cycle only.

LOCATE the phase points of the short business cycle, which will be used to determine the lengths of the successive cycles and of the moving cyclical averages. Mark the phase points on the chart (this is precise), and list them in Table B, column 5, at the proper vertical positions to indicate their dates (this is usually only approximate).

MARK the mid-dates of the cycles on the chart.

TREAT columns 6 to 11 as a block, in so far as the line of entry is concerned. Look first at the mid-date of the particular cycle; suppose it should fall in the third quarter of 1882. Write 1882 Q3 in column 8 (opposite the third quarter of 1882 as listed in column 1). On the same horizontal line in Table B (i. e., opposite 1882, quarter 3) fill in the other figures descriptive of this particular cycle; its name, in column 6, as $p_{1,2}$; the first quarterly date included in the cycle, in column 7; the last quarterly date included, in column 9; the number of quarterly readings included in the cycle, in column 10; and finally the moving cyclical average (whether of the arithmetic or the geometric type), in column 11.

THE above paragraph has been predicated on an assumption or quarterly data. The vertical arrangement in Table B will require double spacing, first, to allow proper placing of the four-quarter moving averages between the dates of column 1; and second, because occasionally two phase points in the annual cycle will fall in the same quarter, and this too requires double spacing.

FOR working from annual data, see Table C. There, too, double spacing is necessary, for the falling of two phase points in the business cycle in the same year occurs frequently, as does the similar collision of mid-dates of cycles.

SHOULD the chart be on semi-log paper, the moving cyclical average (mca) at this stage (working toward SL B) may be made either arithmetic or geometric in type; a little better check with the graphical criteria will be secured from the geometric mean, but it is doubtful whether the slight improvement in accuracy is always worth the extra trouble. See the discussion of the geometric mean below.

PLOT the moving cyclical average (mca) values, each precisely at the mid-date of its cycle, as determined by close measurement on the chart. Connect the mca points in pencil, by sloping straight lines, to form a time polygon. The mca points (and the polygon connecting them) are to serve as guides to the final location of SL B. They will normally lie fairly close to the Tentative SL B, but will furnish a reason for lifting that line in some regions and depressing it in others.

TO draw SL B in an improved location, follow the principles that have already been observed in drawing Tentative SL B, namely, a running equality of areas intercepted above and below SL B, and gentle curvature; and now add the criterion that the line should follow reasonably closely the mca points that have been plotted. The line need not <u>touch</u> each of these mca points; it need merely pass through the area defined by them, following an intermediate path without sacrificing much from the criterion of smoothness, i.e., long radius curvature.

ORDINARILY, this improved location of SL B may stand as final. But two cautions may still be observed:

1) It is necessary to guard against an error that may arise in regions of marked curvature of SL B. This subject will be more thoroughly discussed in locating SL M; only an informal check is suggested here, namely that the operator be sure to go high enough at the peaks in SL B and low enough at the troughs in SL B. He should not smooth too much. The formal check for curvature is more necessary in the next stage, in passing to SL M.

2) It will later become possible to make one more check on the running equality of areas intercepted above and below SL B. In Chapter III will be found a discussion of cycles. The short business cycle is revealed in the record of the ratio of SL A to SL B (Table E and Chart 4). Examination of that cycle gives an opportunity to check once more by the criterion of a running equality of positive and negative areas intercepted. If it is found on the cycle chart that two or three consecutive troughs run too deep (SL A below SL B) to permit the intervening peaks (SL A above SL B) to accomplish the desired running balance of the areas, such a finding would warrant the operator to return to the first or smoothing chart - as Chart 2 - on which the location of SL B had been worked out, and to <u>lower</u> SL B, through the time interval in question. After this correction, when the new values of SL B have been entered in the table, and the new percentage ratios of SL A to SL B have been calculated and plotted on Chart 4, the troughs (SL A below SL B) will be found not so deep, relative to SL B, and the peaks will be found to be higher, relatively, so that the desired running balance of areas will at last have been achieved.

a

WHEN the final line has been determined, read the values of SL B at quarterly intervals, and list them in column 12. So far as the needs of the <u>next</u> stage of smoothing are concerned - in the locating of SL M -- semi-<u>annual</u> values would suffice; but the values are also to be used in determining the standard measures of the short business cycle, by an examination of the quarterly ratios of SL A to SL B; so it is best to take the readings quarterly, at the same dates as those entered in column 1.

Table B. A suggested arrangement for the calculations, from quarterly data, to locate the smoothing lines. (See Table C, and those in Chapter V; there the data are annual rather than quarterly.)

1	2	3	4	5	6	7	8	9	10	11	12	(1)
Date: Year and Quarter	The Quarterly Value	Four Quarter Moving Average	Smoothing Line A	Phase Point (in fluctuation of SL A about Tentative SL B)	Cycle Name	Years and Quarters Included in the Cycle			Number of Quarters Comprised in the Cycle	Moving Cyclical Average (arithmetic or geometric)	Smoothing Line B	(The dates of column 1 may be repeated here)
						Begin	Middle	End				
					(for columns 6 to 11) The mid-date of the cycle is entered in column 8, at a level to correspond with its date. Then the other figures for that cycle are entered on the same level, in columns 6 to 11. The terminal dates of that cycle, columns 7 and 9, show the limiting values of SL A which are to be included in the moving average.							
Double spaced, to permit values in column 3 to be entered between these.	Values in column 3 will be a half-quarter out of phase with columns 1, 2, 4, and 12.		At same dates as in column 1.	Each phase point to be listed at a level which indicates its date.	$p_{1,2}$, $r_{7,8}$, etc.						At same dates as in column 1.	

Note:

Columns 13 to 24 may be added, as in Table Cb, repeating the order of columns 5 to 12, but directed to the locating of SL M. See text on the use of a geometric mean if chart is on semi-log paper. In column 20, enter the values of the Second Approximation to SL M (still subject to correction for curvature). Use a separate work table, as Table Cc, for reiterating the moving average; plot the values of the adjusted moving average; draw final SL M to pass closely through them; read its value at regular intervals and enter in column 24.

Should still another stage of smoothing be undertaken, another block of twelve columns would be needed, as 13 to 24 because of the need again for correcting for curvature. That correction will require a separate work sheet, as described above for the preceding stage.

THE MOVING GEOMETRIC MEAN

IF the chart is on semi-log paper, a moving geometric mean will furnish a "better" guide to the locating of the smoothing line than will a moving average of the arithmetic type. The geometric averages, when plotted as points on the chart, will give a guide that will conform to the other criterion that has been relied upon - the running equality of areas intercepted above and below the smoothing line; this, unfortunately, is not true of arithmetic averages when plotted on semi-log paper. The discrepancy between the two types of average, and consequently the degree of the advantage of the geometric over the arithmetic average, becomes greater when very small and very large items are included within the span of the average. In the cases before us, this is when either: (a) there is a wide scatter in the plotted points, or a wide amplitude in the lower order curve being smoothed; or (b) there is a decided slope in the smoothing line. Should both these circumstances be lacking, the operator may decide to save labor and calculate the arithmetic type of average, despite some slight error that must necessarily result. He may do this with particular confidence in passing from the plotted points to SL A; occasionally in passing from SL A (or from plotted annual values) to SL B; probably never in locating SL M.

THE geometric mean, GM, it will be remembered, is the nth root of the product of n factors, $\sqrt[n]{A \times B \times C \times \dots \times N}$; it may be calculated as the antilog of $\frac{\text{the sum of the logs}}{n}$.

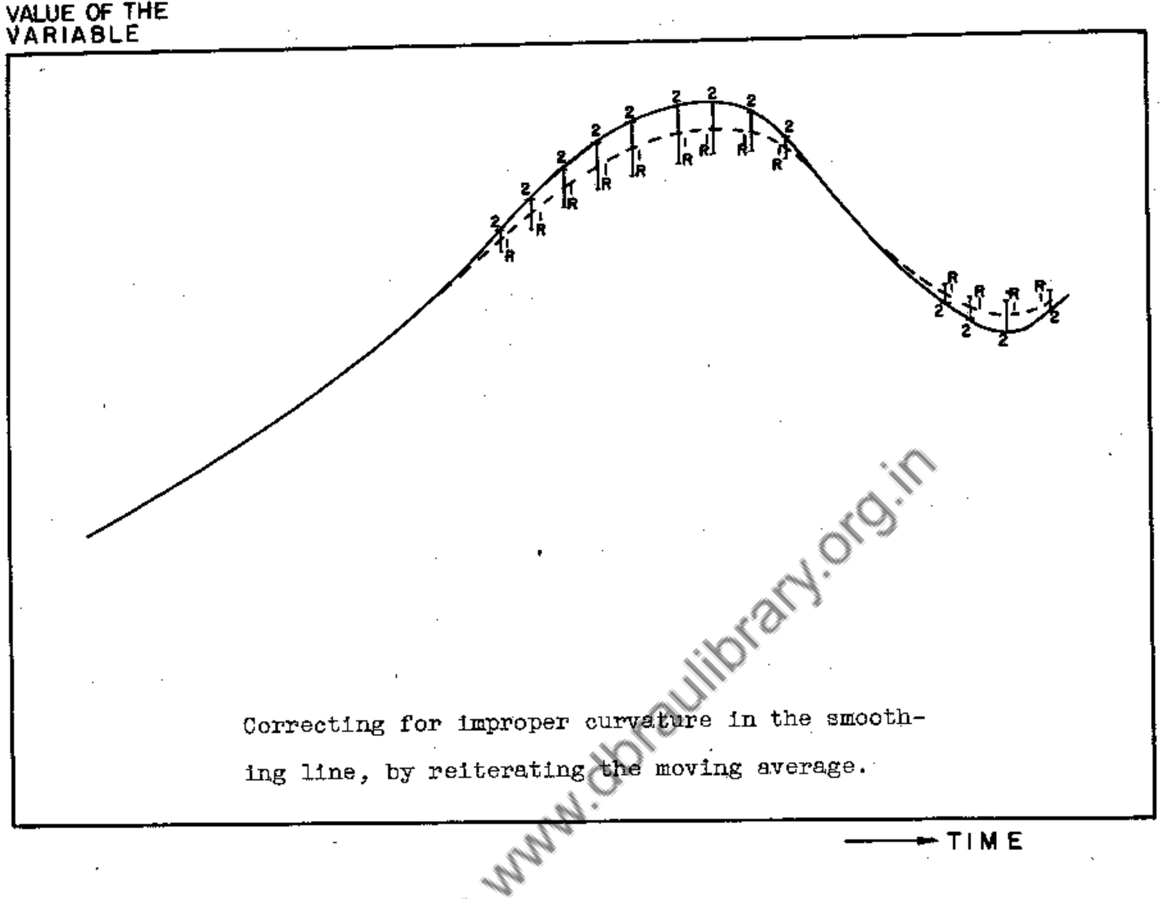
Section 4. LOCATING SMOOTHING LINE M.

FROM SL B, possibly transcribed to a new chart with a condensed time scale, proceed to locate SL M, utilizing a Tentative SL M, phase points, and moving cyclical average (mca). In Table B (continued), columns 13 to 24, enter the calculations; in this stage apply a formal check for curvature; finally enter the values of SL M in column 24 at semi-annual or annual intervals. The calculations for the seven series in Chapter V were arranged in the work table as has just been suggested. But for the presentation or display tables to be printed in this book, some of the columns were compressed into smaller compass.

IF SL M has been well drawn, it will cut through and thereby eliminate the major cycle. There may still be a few "long waves" in SL M, that can be smoothed out in another stage, but if the series is not over sixty years in length, SL M itself should give a very helpful approximation to the secular trend.

THE smoothing line, SL M, in the half-cycle (of SL B about SL M) nearest each end of the chart, should be dotted, so that the reader will appreciate that its location is in some doubt. For a short series, less than about 40 years, it may happen that no mca points will fall in the first eight or ten years, nor in the last ten. This permits the mca check for only a few points in the middle years, and leaves the shape of the dotted ends of SL M to be determined by the criterion of equal areas, and by the trained judgment of the operator.

SKETCH e



Correcting for improper curvature in the smoothing line, by reiterating the moving average.

This correction is required because in periods of marked curvature in the smoothing line, the moving average departs from its "proper" position (which would cut through the middle of the lower order fluctuations), and takes instead an unsuitable position, displaced in the direction of the concavity (i.e. away from the convexity) of the smoothing line.

The sketch omits the lower order line from which the moving averages were calculated; that curve would be more sinuous than those here shown. It also omits the freehand tentative smoothing line, which was used in determining the various cycle lengths in the lower order line; that freehand line was the first approximation to the desired smoothing line. The sketch begins with the moving average values which are labeled "1"; a dashed line connects them; it is the second approximation to the desired smoothing line. Actually, because of its displacement in the direction of concavity, the second approximation may be rather badly out of place; yet it will aid in locating the final, satisfactory line.

Read the values of the dashed line (at the same dates as those of the lower order line, previously used in calculating the moving average points marked "1") and enter them in a work table, not here shown. Using these values of the dashed line, calculate a new set of moving average values; it is this process which is called reiterating the moving average. The "R" points have been plotted on this chart, though they would never be plotted on a working chart, as they are not wanted in themselves - only their differences from the points marked "1". That difference or discrepancy is the desired error due to curvature. Project that difference in the opposite direction from point "1"; it will be found that the projection is in the direction of convexity in the smoothing curves. Thereby secure point "2", the desired final guide to the smoothing line, which has been corrected for the curvature in the smoothing line itself. The line through the points marked "2" is here made a solid line. It will properly cut through the middle of the lower order fluctuations.

CURVATURE: A SYSTEMATIC ERROR IN MOVING AVERAGES; ITS CORRECTION BY REITERATION OF THE MOVING AVERAGE.

MOVING averages exhibit a systematic error when fitted to a curve with sharp curvature. This is true whether they are of the ordinary form with constant period, or moving cyclical averages with changing lengths. As soon as the nature of this error is once understood, its correction may be undertaken.

SUPPOSE that SL B has been established, and that one is working toward the location of SL M, to eliminate the major cycle. He has drawn tentative SL M, as on Chart 26, and observes a peak in it in the vicinity of the year 1925. Because of the curvature at this peak, moving average values (labelled #1) in the vicinity of 1925 will be "too small"; that is, when plotted on the chart they will stand too low to accomplish their intended purpose, which is to cut through the middle of the major cycle fluctuations in SL B. A smoothing line standing so low would not achieve the complete segregation of the several orders of cycles; it would go beyond its intended function, which is solely to smooth out the major cycle, and it would contribute something undesired toward smoothing the "long wave" as well. This overly-smooth line would make an unsuitable base of reference for the study of the major cycle, (Chapter III) and would also be unsuitable for the study of "long waves", should that stage of cycle analysis be undertaken.

TO begin the process of correcting for the error of over-smoothing, draw a second approximation to SL M to touch these dubious (#1) moving average points. This second approximation line is designed to serve as a base (1)to find by how much the moving averages near 1925 are too low, and consequently to correct for the error. Read values from the second approximation line at regular intervals, at the same dates as the SL B values that were used in the mca process previously undertaken (which gave the mca points through which this second approximation to SL M has been drawn), and enter them in column 20 of Table B.These regularly spaced readings from the second approximation to SL M are to be used to calculate new or "reiterated" moving cyclical average values; use precisely the same cycle lengths for the calculation of the reiterated moving cyclical averages as were used for the first set.

BECAUSE of the curvature in SL M (as shown both in Tentative SL M and in the second approximation through the #1 mca points), these second or reiterated moving averages will be still further too small, that is, if they were plotted, they would lie below the second approximation line. But now we can compare the second approximation line with the reiterated values. For each reiterated mca, determine the difference between its value and the value of the second approximation line at the same date. This difference may be called the error in the moving average caused by curvature; to correct, the first mca may be increased by this amount, to secure the desired guide point.

FOR example, in Tables Cb and Cc, one of the original mca points ($t_{3,4}$, column 19) fell in 1925, and its value was 10,800 deeds per year; the reiteration of the moving average showed a new mca (column 21) of 10344, which shows a departure (column 22) from the second approximation to SL M of 456 deeds per year. 456 is the error due to curvature, and should be

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(1) In Chapter V. Table F, two bases of reference are combined, in a rather free compromise; the second approximation line here called for, and also the first set of mca values. (The other tables in Section VII refer only to one base of reference.)

taken as the correction; 10800 plus 456 gives 11,256, the adjusted mca value for 1925. The value 11,256 should be taken as the graphical guide to the final SL M.

THE amount of error in the value of the moving average, due to curvature, is greater where the curvature in the smoothing line is sharper, and where the particular cycle in the lower order curve about that smoothing line is longer; the error and the amount of the correction, consequently, will be found to vary from one mca point to the next.

THE above discussion has related to 1925, a peak. At a trough, as that of 1895 on Chart 8, the error due to curvature causes the first set of moving averages to be too high; the reiteration gives a value still higher; the correction arrived at by subtracting the second approximation line from the reiterated mca should be subtracted from the value of the first mca, to obtain the adjusted mca, the desired graphical guide to the final SL M.

IT has been the practice of the writer to make only an informal correction for curvature in the short business cycle - in locating SL B. He usually makes a formal correction only in the next stage, and then, principally, through the years in which distinct curvature is evident on the chart. See the smoothing in Table C, parts a, b, and c, and Charts 2 and 3.

USE OF SMOOTHING LINE M AS A GUIDE TO A MATHEMATICALLY FITTED TREND.

FROM an inspection of SL M, the operator may conclude that it resembles some particular mathematical curve (such as a straight line, a second degree parabola, a compound interest curve, or a logistic curve). He may then choose to go back to the original data and by mathematical or total process, to fit a curve of the type selected. Such procedure might be thought to be a rejection of the SL M, and therefore of the method of successive smoothings. But even in this case, the method of successive smoothings, which has been relatively easily applied, will at least have given a basis for determining which type of trend to fit; moreover, it will probably have furnished, fairly closely, the parameters of the equation. (See in Chapter V how a group of logistic curves previously fitted by Kuznets are tested by the method of successive smoothings.) Also, as will be developed in Chapter III, the method of successive smoothings will furnish an excellent analysis of the cyclical components of the time series.

Section 5. SAN FRANCISCO REAL ESTATE ACTIVITY, 1867 TO 1940

Application of Two Stages of Smoothing to Annual Data

(a discussion of Table C and Charts 2 and 3)

THIS illustration begins with annual data, and consequently there is no SL A to be drawn, and no seasonal analysis to be made. Yet, for simplicity in cross-reference, the columns have been numbered as in Table B. Table C and Charts 2 and 3 were prepared together, as will be described.

FROM the tabulated values of the annual figures, in Table Ca, column 2, a time polygon was drawn on Chart 2. Tentative Smoothing Line B was drawn in pencil, to cut through that time polygon (the reader will find parts of this tentative line dashed, but other parts merge with the solid line which marks the final location of SL B). The fluctuations of the time polygon about Tentative Smoothing Line B, were observed, and the phase points were marked on the chart and entered in the table (column 5).

THE figures in the next six columns find their vertical position determined by the date of the middle of the cycle (entered in column 8); these mid-points were marked on the chart and entered in the table. Columns 6, 7, 9, 10, and 11 were then filled in. (Note that in column 5 the phase points are entered in the vertical position which corresponds to the year in which they fall; and that in columns 6 to 11 the names and measures of the cycles are entered in the vertical position which corresponds to the mid-date of the respective cycle, entered in column 8). The tabulated length of the cycle in years (column 10) is not intended as a precise measure of the elapsed time, or it would be necessary to include fractions of years; rather, it is an enumeration or count of the number of annual figures to be included in the calculation of the moving cyclical average. The precise mid-date of the cycle is not available in the table, as the writer does not consider it to be of value for any purpose other than the graphical application; if another operator wishes, he may, of course, make this information precise in the table.

AN average calculated for a cycle containing but one or two annual values, is not very representative, and is but a poor guide to the desired smoothing line. No formal step was taken here to secure a more representative figure, which might afford a better guide, but in half of these cases such a step would be feasible. For a cycle between two "r" points, or a cycle between two "f" points, one added annual value could be included at each end, and the new mca could be plotted, along with the first mca; both could be used as guides. But this procedure is not suitable for a cycle bounded by two "p" points or by two "t" points; in the one case, two large values would be added, and the resulting average would be too great to serve as a good guide; in the second case, the two small values added would warp the average downward.

WHEN all the mca points had been plotted, Tentative SL [illegible] mca points were inspected, and the final SL B was drawn. At this step, the various criteria were reviewed: a running equality of areas intercepted, smoothness, etc.; and in addition the new criterion of nearness to the mca points was considered. Care was exercised to go high enough at the peaks in SL B and low enough at the troughs - that is to say, at points of noticeable curvature the final SL B was caused to stay well out in the convex direction from the center of curvature. The chart and table were kept in pencil for another final check, which became available when Chart 4 had been drawn. There, a new view could be obtained of the desired running equality of areas intercepted between the time polygon and the final SL B. Finally, the accepted location of SL B was inked in as a solid line with dotted ends. Annual values of SL B were read from the chart and entered in the table (column 12).

ATTENTION has been called to the fact that the terminal portions of the "final" SL B are dotted, because of uncertainty as to direction and curvature. When forecasting is undertaken, and a definite attitude toward the future is assumed by the operator, that attitude will be reflected in reforming the dotted end of SL B. See, on Chart 6, how the dotted end comes in for substantial modification.

THE next stage of smoothing was then begun, in Table Cb and on Chart 3. Through the fluctuations in SL B, tentative SL M was drawn, with care to the criteria of a running equality of areas, and smoothness. The phase points were marked on a horizontal line near the bottom of the chart, the mid-points of the cycles on the next line below. The dates of the phase points and of the mid-points of the cycles were read from the chart and their names were entered in the table, each at the proper level to indicate its date. Moving cyclical averages were calculated, and plotted at the mid-dates of the cycles. The second approximation to SL M was drawn through the mca points.

IN Table Cc, this second approximation to SL M was submitted to a correction for curvature, involving a reiteration of the mca calculation. The necessary data were taken from Table Cb, and given the same column numbers; the correction for curvature was determined; and the original mca values were adjusted. Final SL M was then drawn on Chart 3, and its values entered in the last column of Table Cb, which column has been numbered 24 to indicate that it follows Table Cc.

Table Ca. SAN FRANCISCO REAL ESTATE ACTIVITY 1867 TO 1940

The First Stage of Smoothing: Locating SL B

(columns numbered to correspond with Table B)

1	2(or 4)	5	6	7	8	9	10	11	12
Year	Number of Deeds	Phase Point	Cycle	Yearly Figures Included in the Cycle			Length in Years	Moving Cyclical Average	Smoothing Line B
				Begin	Middle	End			
1867	5556	r_1							(6000)
68	6724								(5600)
69	6908	p_1							(5250)
1870	4677	f_1							4950
71	4016		$r_{1,2}$	1868	1871	1874	7	4710	4600
72	3657		$p_{1,2}$	1869	1872	1875	7	4394	4250
73	3143	t_1	$f_{1,2}$	1870	1873	1876	7	3957	3900
74	3854	r_2							3650
75	4512	p_2							3350
76	3840								3150
			$t_{1,2}$	1873	1876-77	1880	8	3198	
77	3085	f_2							2950
78	2610								2850
79	2217		$r_{2,3}$	1875	1879	1883	9	2883	2750
1880	2331	t_2	$p_{2,3}$	1876	1880	1884	9	2812	2700
81	2277		$f_{2,3}$	1877	1881	1885	9	2789	2750
82	2385								2850
83	2687	r_3	$t_{2,3}$	1881	1883	1886	6	2993	2950
84	3874	p_3							3100
85	3633	f_3	$r_{3,4}$	1884	1885	1886	3	3536	3450
86	3101	t_3	$p_{3,4}$	1885	1886	1887	3	3911	3950
		r_4	$f_{3,4}$	1886	1886	1887	2	4050	
87	4998	p_4	$t_{3,4}$	1886	1887	1888	3	4488	4700
		f_4	$r_{4,5}$	1887	1888	1888	2	5182	
88	5366	t_4	$p_{4,5}$	1888	1888	1889	2	6051	5650
		r_5	$f_{4,5}$	1888	1888-89	1889	2	6051	
89	6736	p_5	$t_{4,5}$	1889	1889	1890	2	6708	6550
		f_5	$r_{5,6}$	1889	1889-90	1890	2	6708	
1890	6680	t_5	$p_{5,6}$	1890	1890	1890	1	6680	6900

(Table Ca is continued on next page)

Table Ca (continued) San Francisco Real Estate Activity

First Stage of Smoothing.

1 Year	2(or 4) Number of Deeds	5 Phase Point	6 Cycle	7	8 Yearly Figures Included in the Cycle	9	10 Length	11 mca	12 SL B
				Begin	Middle	End			
		r_6	$f_{5,6}$	1890	1890-91	1891	2	6718	
1891	6757	p_6							6250
92	4958	f_6	$t_{5,6}$	1890	1892	1893	4	5628	5250
			$r_{6,7}$	1891	1892	1894	4	4809	
93	4117	t_6	$p_{6,7}$	1892	1893	1895	4	3998	4250
94	3404	r_7	$f_{6,7}$	1892	1894	1896	5	3852	3500
95	3515		$t_{6,7}$	1894	1895	1897	4	3100	3050
96	3267	p_7	$r_{7,8}$	1895	1896	1898	4	2910	2800
97	2215	f_7	$p_{7,8}$	1896	1897	1898	3	2709	2650
		t_7							
98	2645	r_8	$f_{7,8}$	1897	1898	1899	3	2834	2700
99	3053	p_8	$t_{7,8}$	1898	1899	1900	3	2986	2950
		f_8							
1900	3259	t_8	$r_{8,9}$	1899	1900	1901	3	3524	3550
			$p_{8,9}$	1899	1900-01	1902	4	4098	
01	4261	r_9	$f_{8,9}$	1900	1901	1902	3	4444	4400
02	5813	p_9	$t_{8,9}$	1901	1902	1903	3	5480	5500
03	6365	f_9	$r_{9,10}$	1902	1903	1904	3	6417	6800
04	7073	t_9	$p_{9,10}$	1903	1904	1905	3	7670	7900
		r_{10}							
05	9572	p_{10}	$f_{9,10}$	1903	1905	1907	5	8032	8300
06	8947		$t_{9,10}$	1904	1906	1908	5	8243	8450
07	8204	f_{10}	$r_{10,11}$	1905	1907	1909	5	8423	8400
08	7418	t_{10}	$p_{10,11}$	1906	1908	1910	5	8214	8300
09	7972	r_{11}							8150
1910	8528		$f_{10,11}$	1908	1910	1912	5	7996	8000
11	8162	p_{11}	$t_{10,11}$	1909	1911	1914	6	7572	7650
12	7900		$r_{11,12}$	1909	1912	1915	7	7281	7300
13	6702	f_{11}							6900
			$p_{11,12}$	1911	1913-14	1916	6	6846	
14	6171								6500
1915	5533	t_{11}	$f_{11,12}$	1913	1915	1917	5	6194	6100

(Table Ca is concluded on next page)

Table Ca (concluded) San Francisco Real Estate Activity

First Stage of Smoothing.

1 Year	2(or 4) Number of Deeds	5 Phase Point	6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Middle	9 End	10 Length	11 mcs	12 SL B
1916	6610	r_{12} p_{12}	$t_{11,12}$	1915	1916	1917	3	6032	5850
17	5952	f_{12}	$r_{12,13}$	1916	1917	1919	4	6130	5850
18	4818	t_{12}	$p_{12,13}$	1917	1918	1920	4	6778	6200
19	7138	r_{13}	$f_{12,13}$	1918	1919	1920	3	7053	7300
			$t_{12,13}$	1918	1919-20	1921	4	7724	
1920	9203	p_{13}							8800
		f_{13}	$r_{13,14}$	1920	1920-21	1922	3	10486	
21	9736	t_{13}							10500
			$p_{13,14}$	1920	1922	1923	4	11600	
22	12519	r_{14}	$f_{13,14}$	1921	1922	1923	3	12365	12700
			$t_{13,14}$	1922	1922-23	1923	2	13730	
23	14940	p_{14}	$r_{14,15}$	1922	1923	1924	3	14370	14600
		f_{14}							
24	15650	t_{14}	$p_{14,15}$	1923	1924	1925	3	16291	16300
		r_{15}							
25	18282	p_{15}	$f_{14,15}$	1924	1925	1926	3	16825	16850
26	16543								15600
			$t_{14,15}$	1924	1926-27	1928	5	14883	
27	12960	f_{15}	$r_{15,16}$	1925	1927	1930	6	12785	13500
28	10980		$p_{15,16}$	1926	1928	1931	7	11162	11600
29	9416	t_{15}							10000
			$f_{15,16}$	1928	1929-30	1932	5	8922	
1930	8528	r_{16}							8850
31	8548	p_{16}	$t_{15,16}$	1929	1931	1934	6	7193	7600
32	7139	f_{16}							6500
33	5158		$r_{16,17}$	1931	1933	1935	5	6210	5900
34	4368	t_{16}	$p_{16,17}$	1932	1934	1936	5	6091	5900
35	5839		$f_{16,17}$	1933	1935	1937	5	6403	6300
36	7949	r_{17}	$t_{16,17}$	1934	1936	1938	5	7009	7150
37	8701	p_{17}							8000
38	8190	f_{17}	$r_{17,18}$	1936	1938	1939	4	8425	(8800)
		t_{17}							
39	8859								(9500)
1940	10658	r_{18}							(10350)

Table Cb. SAN FRANCISCO REAL ESTATE ACTIVITY

1867 TO 1940

The Second Stage of Smoothing: Locating SL M

(columns numbered to correspond with Table B)

(See the Tables in Chapter V, for a more condensed arrangement)

1	12	13	14	15	16	17	18	19	20	24
Year	SL B	Phase Point	Cycle	Yearly Figures Included in the Cycle Begin	Yearly Figures Included in the Cycle Middle	Yearly Figures Included in the Cycle End	Length in Years	mca	Second Approx. to SL M	Final SL M (See Table Cc)
1867	(6000)								(4000)	(4000)
68	(5600)								(4000)	(4000)
69	(5250)								(4000)	(4000)
1870	4950								(4000)	(4000)
71	4600								(4000)	(4000)
72	4250	f_1							(4000)	(4000)
73	3900								(4000)	(4000)
74	3650								(4000)	(4000)
75	3350								(4000)	(4000)
76	3150								(4000)	(4000)
77	2950								(4000)	(4000)
78	2850								(4000)	(4000)
79	2750								(4050)	(4000)
1880	2700								(4050)	(4000)
81	2750	t_1							(4050)	(4000)
82	2850		$f_{1,2}$	1872	1882	1892	21	4093	(4100)	4000
83	3000								4100	4010
84	3100								4150	4020
85	3450								4150	4030
86	3950								4200	4040
87	4700	r_1							4200	4060
88	5650								4200	4075
89	6550		$t_{1,2}$	1881	1889	1898	18	4186	4250	4100
1890	6900	p_1							4250	4130

(Table Cb is continued on next page)

Table Cb (continued) San Francisco Real Estate Activity

Second Stage of Smoothing

1	12	13	14	15	16	17	18	19	20	24
Year	SL B	Phase Point	Cycle	Yearly Figures Included in the Cycle			Length	mca	Second Approx. to SL M	Final SL M
				Begin	Middle	End				
1891	6250								4300	4160
92	5250								4350	4200
93	4250	f_2							4400	4250
94	3500								4500	4320
			$r_{1,2}$	1889	1894-95	1902	14	4414	(4550)	
95	3050								4600	4410
96	2800								4700	4510
97	2650								4800	4630
			$p_{1,2}$	1890	1897-98	1905	16	4797	(4875)	
98	2700	t_2							4950	4760
99	2950								5050	4900
1900	3550								5200	5050
01	4400								5350	5210
02	5500	r_2	$f_{2,3}$	1893	1902	1911	19	5647	5500	5400
03	6800								5650	5590
04	7900								5850	5690
05	8300	p_2							6050	6000
06	8450								6250	6200
07	8400								6500	6460
08	8300		$t_{2,3}$	1898	1908	1917	20	6475	6700	6700
09	8150								6950	6980
1910	8000								7250	7250
11	7650								7600	7580
		f_3	$r_{2,3}$	1903	1911-12	1920	18	7375	(7750)	
12	7300								7900	7900
13	6900								8200	8200
14	6500								8500	8500
1915	6100		$p_{2,3}$	1906	1915	1925	20	9035	8850	8900

(Table Cb is concluded on next page)

Table Cb (concluded) San Francisco Real Estate Activity

Second Stage of Smoothing

1	12	13	14	15	16	17	18	19	20	24
Year	SL B	Phase Point	Cycle	Yearly Figures Included in the Cycle			Length	mca	Second Approx to SL M	Final SL M
				Begin	Middle	End				
1916	5850								9100	9230
17	5850	t_3							9350	9520
18	6200								9600	9840
19	7300								9900	10150
1920	8800		$f_{3,4}$	1912	1920	1928	17	10144	10100	10400
21	10500	r_3							10250	10670
22	12700								10450	10900
23	14600								10550	11070
24	16300								10650	11200
25	16800	p_3	$t_{3,4}$	1918	1925	1933	16	10800	10800	11210
26	15600								10750	11160
27	13500								10700	11030
28	11600								10600	10880
29	11000	f_4							10500	10710
			$r_{3,4}$	1921	1929-30	1938	18	10369	(10425)	
1930	8850								10350	10520
31	7600								10250	10330
32	6500								(10100)	(10170)
33	5900	t_4							(9950)	(9990)
34	5900								(9800)	(9810)
35	6300								(9600)	(9600)
36	7150								(9450)	(9450)
37	8000								(9250)	(9250)
38	(8800)	r_4							(9100)	(9100)
39	(9500)								(8900)	(8900)
1940	(10350)								(8700)	(8700)

Table Cc. SAN FRANCISCO REAL ESTATE ACTIVITY, 1867 TO 1940

The second stage of smoothing, continued: Correction for curvature.

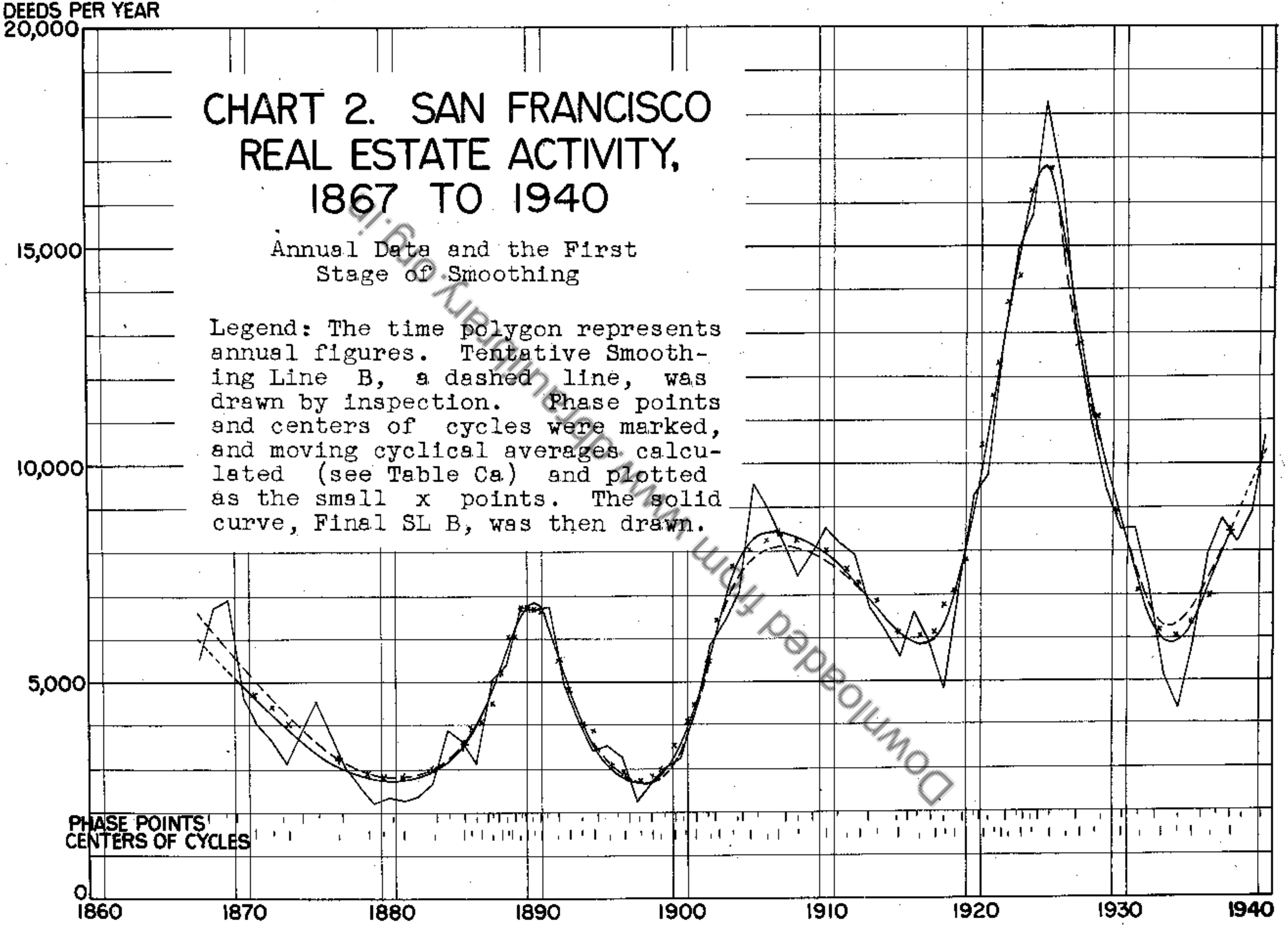
(Columns 14 to 20 repeated, in condensed form, from Table Cb)

14	15	16 Yearly Figures in the Cycle	17	18 Length in years	19 m.c.s. Based on SL B	20 Second Approx. to SL M	21 Reiterated m. c. s., based on Second Approx. to SL M	22, Curvature Correction 20-21	23 Adjusted m. c. s. 19+22
Cycle	Begin	Middle	End						
$f_{1,2}$	1872	1882	1892	21	4093	4100	4114	-14	4079
$t_{1,2}$	1881	1889	1898	18	4186	4250	4347	-97	4089
$r_{1,2}$	1889	1894-95	1902	14	4414	4550	4729	-179	4235
$p_{1,2}$	1890	1897-98	1905	16	4797	4875	4969	- 94	4703
$f_{2,3}$	1893	1902	1911	19	5647	5500	5676	-176	5471
$t_{2,3}$	1898	1908	1917	20	6475	6700	6838	-138	6337
$r_{2,3}$	1903	1911-12	1920	18	7375	7750	7795	- 45	7330
$p_{2,3}$	1906	1915	1925	20	9035	8850	8773	+ 77	9112
$f_{3,4}$	1912	1920	1928	17	10144	10100	9780	+320	10464
$t_{3,4}$	1918	1925	1933	16	10800	10800	10344	+456	11256
$r_{3,4}$	1921	1929-30	1938	18	10369	10425	10172	+253	10622

The final column, number 24, is in Table Cb.

CHART 2. SAN FRANCISCO REAL ESTATE ACTIVITY, 1867 TO 1940

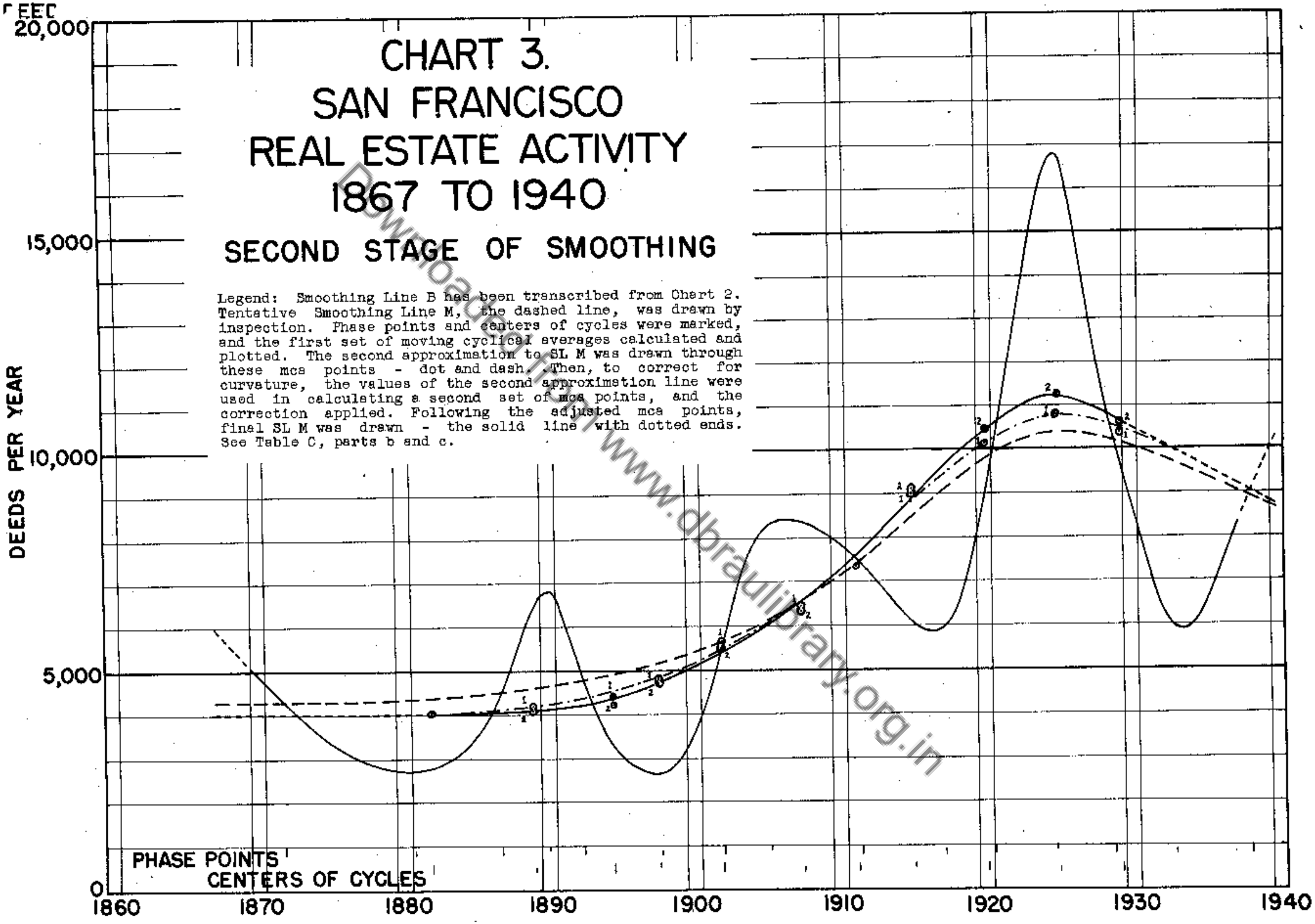
Annual Data and the First Stage of Smoothing



Legend: The time polygon represents annual figures. Tentative Smoothing Line B, a dashed line, was drawn by inspection. Phase points and centers of cycles were marked, and moving cyclical averages calculated (see Table Ca) and plotted as the small x points. The solid curve, Final SL B, was then drawn.

CHART 3.
SAN FRANCISCO
REAL ESTATE ACTIVITY
1867 TO 1940

SECOND STAGE OF SMOOTHING



Legend: Smoothing Line B has been transcribed from Chart 2. Tentative Smoothing Line M, the dashed line, was drawn by inspection. Phase points and centers of cycles were marked, and the first set of moving cyclical averages calculated and plotted. The second approximation to SL M was drawn through these mcs points - dot and dash. Then, to correct for curvature, the values of the second approximation line were used in calculating a second set of mcs points, and the correction applied. Following the adjusted mcs points, final SL M was drawn - the solid line with dotted ends. See Table C, parts b and c.

CHAPTER III.

THE CYCLES

Section 1. CHARACTERISTICS OF THE CYCLES.

IT will be recalled that to locate the smoothing lines is but the first of three major tasks in studying time series by the method of smoothing by stages. The second objective is to organize the information concerning the several orders of cycles. Prior to this organizing, one can examine the historical record of each order of cycle; this is of course an essential and valuable step. But it is important to go beyond this mere viewing of the cycle; the method of smoothing by stages makes it possible to calculate standard measures of each order of cycle. Armed with these measures, one can furnish an extensive and meaningful description of the time series, can compare the particular series with others, and can make a systematic forecast. (Comparison and correlation of series, and forecasting, will be treated briefly in Chapter IV.)

MEASURES OF THE ANNUAL CYCLE.

THE relationship of actual monthly or quarterly data to SL A, constitutes the annual cycle. The record of this ratio is completely freed from all elements of the short business cycle, the major cycle, and the trend. The standard pattern of this order of fluctuation is commonly called the seasonal pattern, or the four quarterly (or twelve monthly) indexes. The seasonal pattern may be calculated from the record of the ratios of actual to SL A, in the same way that it is usually calculated from the ratios to the moving average. Subsequently, as in that method, after the standard seasonal pattern for the entire record has been determined, the record may be broken into parts, and separate calculations made for the early years and for the late years, in order to discover changes in the seasonal pattern.

ONE may calculate the typical date in the year at which the seasonal peak occurs, the typical date of the trough, the typical "r" date, and the typical "f" date.

THE typical percentage deviation of actual from SL A at the peak is another useful measure; and the typical percentage deviation of actual from SL A at the trough.

FINALLY, the amplitude of the annual cycle may be calculated, that is, the standard deviation (sd) of the actual values from SL A, measured in per cent.

WITH these measures, one can draw the annual cycle as it would appear in the typical or usual year, and can compare it with similar typical annual cycles calculated for other series. (No such analysis of the annual cycle is offered among the illustrations in this book.)

MEASURES OF THE SHORT BUSINESS CYCLE.

THE relationship of actual annual data (or of SL A) to SL B, will give the movement which is called the short business cycle. By the smoothing process by which SL A was located, the ratio of SL A to SL B has been completely freed from shorter period impulses, which have been taken into the annual cycle. It is also freed from the major cycle and all longer movements, for these are deferred to later stages of the analysis; these long movements are present both in the actual annual data and SL B (or in SL A and in SL B), and they consequently do not appear in the relationship between those two lines (they cancel out in the numerator and denominator of the ratio $\frac{\text{annual data}}{\text{SL B}}$ or $\frac{\text{SL A}}{\text{SL B}}$).

THE amplitude or standard deviation (sd) may be calculated.

SEVERAL time-lengths will aid one to construct the typical cycle of this order; the four typical phase lengths, pf, ft, tr, and rp; and also their sum, the typical over-all length of the short business cycle. Table Db will show an arrangement for this calculation; it may be seen also in Table E and on Charts 4 and 5.

THE typical percentage ratio of $\frac{\text{SL A}}{\text{SL B}}$ (or $\frac{\text{annual data}}{\text{SL B}}$) at the peak, and the typical percentage ratio at the trough may be found by simple averaging.

WITH these standard measures of the short business cycle, one can draw that cycle (Chart 5), and can make a forecast (Chart 6).

MEASURES OF THE MAJOR CYCLE.

THE relationship of SL B to SL M will give the history of the major cycle. From that history, standard measures of the major cycle may be determined: the amplitude or standard deviation (sd), the typical ratios of SL B to SL M at the peak and at the trough, the typical length of the cycle as a whole, and the typical length of each of the four phases. The calculation is arranged in Table Dc.

AGAIN, as in the annual cycle and the short business cycle, this information enables the operator to draw and to describe the major cycle, and to use its shape in a forecast.

FURTHER lines of study may be suggested:

a) The study of the history of each order of cycle may be carried forward by differences, instead of by ratios. The method of differences is recommended if some of the values approach zero, and it is required if they pass into negative values - as would be the case in a study of gold movements into a country, and of other incremental variables.

b) In analyzing each order of cycle, the student may be interested not merely to determine the typical value of each measure, but to examine the full range or distribution of the values of that measure. He may be concerned to discover systematic changes in that measure with the passage of time -- or during particular phases of a longer movement than this particular cycle. (See W. C. Mitchell, Business Cycles, chapter 3, on the contribution of statistics.) For example he

Table Da. Arrangement for the Calculation of the Standard Measures of the Seasonal or Annual Cycle

1 Date (quarter or month)	2 Percentage ratio $100 \times \frac{\text{datum}}{\text{SL A}}$ (in %)	3 Deviation of ratio from 100% (in %)	4 Deviation squared

5. From the sum of the figures in column 4, calculate sd, the amplitude of the fluctuation: $sd = \sqrt{\frac{\Sigma\,(d^2)}{n}}$

6. From column 2, calculate the seasonal indexes:

	Ratio, datum to SL A			
	Q_1	Q_2	Q_3	Q_4
1920	.....%	.....%	.....%	.....%
1921	.....	.....	.....	.....
1922	.....	.....	.....	.....
etc.	.....	.....	.....	.....
Average ratio	.....%	.....%	.....%	.....%

Adjust to bring the total of these four average ratios to 400%, and so get the seasonal indexes.

7. Calculate the typical date in the year for each phase point. Take information from a chart like Chart 2. Possibly smooth monthly data first, by a 3-month moving average.

	Time, measured from January 1			
	p	f	r	t
1920	.....	.....	.....	.....
1921	.....	.....	.....	.....
1922	.....	.....	.....	.....
etc.	.....	.....	.....	.....
Typical or average date in the year	.....	.....	.....	.....

8. Calculate the typical percentage deviation of actual from SL A, at peak and at trough.

	Deviation of actual from SL A	
	at peak	at trough
1920	......%	......%
1921	......	......
1922	......	......
etc.	......	......
Typical or average deviation	......% at peak	......% at trough

Table Db. Arrangement for the Calculation of the Standard Measures of the Short Business Cycle

1 Date (Year or Quarter)	2 Percentage Ratio $100 \times \frac{\text{datum}}{\text{SL B}}$, or $100 \times \frac{\text{SL A}}{\text{SL B}}$ (corresponds to column 25 in the cycle tables in Chapter V)	3 Deviation of Ratio from 100% (in %) (corresponds to column 26 in Chapter V)	4 Deviation squared (corresponds to column 27 in Chapter V)

5. From the sum of the figures in column 4, calculate sd, the amplitude of the fluctuation: $sd = \sqrt{\frac{\Sigma (d^2)}{n}}$

(Here a step must be omitted, that appeared as No. 6 in Table Da, the first order analysis.)

7. Calculate the typical length of each phase or quarter of the short business cycle. (corresponds to step 28 in the cycle tables in Chapter V)

	Length of phase in years and fractions			
Phase or Quarter	pf	ft	tr	rp
First cycle	....yrs.	....yrs.	....yrs.	....yrs.
Second cycle	....	....	....	....
Third cycle	....	....	....	....
etc.				
Typical or average length	....yrs.	....yrs.	....yrs.	....yrs.

The sum of these four typical phase lengths is the over-all length of the typical short business cycle.

8. Calculate the typical percentage deviation of SL A from SL B, at the peak and at the trough. (corresponds to step 29 in the cycle tables in Chapter V)

	Deviation at peak	Deviation at trough
First cycle	.......%	.......%
Second cycle	....	....
Third cycle	....	....
etc.		
Typical or average deviation	.......%	.......%

Table Dc. Arrangement for the Calculation of the Standard Measures of the Major Cycle

(Note that the details here are almost the same as those in Table Db, save that here the attention is on the ratio of SL B to SL M, and the values are presumed to be listed at annual intervals.)

1 Year	2 Percentage Ratio $100 \times \frac{SL\ B}{SL\ M}$	3 Deviation of Ratio from 100% (in %)	4 Deviation squared
	(corresponds to column 30 in the cycle tables in Chapter V)	(corresponds to column 31 in Chapter V)	(corresponds to column 32 in Chapter V)

5. From the sum of the figures in column 4, calculate sd, the amplitude of the fluctuation: $sd = \sqrt{\frac{\Sigma(d^2)}{n}}$

(Here, as in Table Db, a step must be omitted)

7. Calculate the typical length of each phase or quarter of the major cycle. (corresponds to step 33 in the cycle tables in Chapter V)

Length of phase in years and fractions

Phase or Quarter	pf	ft	tr	rp
First cycle	....yrs.	....yrs.	....yrs.	....yrs.
Second cycle	....	....	....	....
Third cycle	....	....	....	....
etc.				
Typical or average length	....yrs.	....yrs.	....yrs.	....yrs.

The sum of these four typical phase lengths is the over-all length of the typical major cycle.

8. Calculate the typical percentage deviation of SL B from SL M at the peak and at the trough. (corresponds to step 34 in the cycle tables in Chapter V)

	Deviation at peak	Deviation at trough
First cycle	......%	......%
Second cycle	......	......
Third cycle	......	......
etc.		
Typical or average deviation	......%	......%

may study the seasonal pattern during major booms, or the short business cycles during major depressions.

EXPLANATION OF A USE OF STRAIGHT LINES.

ON Chart 2, the actual annual data were plotted; those values are discrete (not continuous). They were then connected by straight lines to form a time polygon; but those straight lines have no important theoretical meaning; they serve merely as connecting links. On Chart 4, the ratios of $\frac{\text{actual}}{\text{SL B}}$ have been plotted for every year. To be mathematically consistent with the straight lines connecting the data on Chart 2, these points on Chart 4 should be connected by segments of curves, whereas they too have been connected by straight lines. But these annual ratios are discrete also, and the straight lines connecting them to form the time polygon have no important theoretical meaning. Moreover, the curvature (if drawn) in the short segmentary lines would not be sharp---on the whole they would be nearly indistinguishable from straight lines, and the work would be laborious.

Section 2. SAN FRANCISCO REAL ESTATE ACTIVITY, 1867 TO 1940

Analysis of Two Orders of Cycles (a discussion of Table E and Chart 4)

THE percentage ratio of the actual number of deeds in each year to the corresponding value of SL B, $\frac{100 \text{ x data}}{\text{SL B}}$, is entered in Table Ea and plotted on Chart 4. These ratios show the history of the short business cycle. Similarly, in Table Eb, and on Chart 4, there will be seen the history of the major cycle as shown by the ratio $\frac{100 \text{ x SL B}}{\text{SL M}}$ (the multiplier 100 is introduced to transform the simple ratio to a percentage ratio).

IT will be seen that in each stage, the terminal half-cycle has been omitted from this part of the study. This is because the smoothing line in the terminal half-cycle has not a firmly established value, hence those uncertain ratios may not properly be used in determining the standard measures of the cycle.

THE calculation of sd, the amplitude of the cycle, needs no description.

THE time lengths of the successive phases were read from Chart 2, for the calculations in Table Ea; and from Chart 3, for the calculations in Table Eb. They are averaged in simple fashion.

THE percentage deviations at peak and at trough were taken from the listings in Tables Ea and Eb respectively.

SURPRISINGLY, in this particular series, the average or typical deviation from SL B at the peak (and that at the trough) of the short cycle, is less than the standard deviation of all the data about SL B. This is due to a difference in the two methods of calculation. The sd is calculated by squaring all the deviations before striking an average; the few extreme deviations exert a great influence on the result. On the other hand, the typical peak (and trough) are calculated by averaging the first powers of the deviations at the several peaks (or troughs); the few extreme cycles here exert less influence.

Table Ea. SAN FRANCISCO REAL ESTATE ACTIVITY

Calculation of Standard Measures of the Short Business Cycle, from the Record 1867 to 1940:

the amplitude or standard deviation;
the typical lengths of the phases or quarters of the cycle;
the typical ordinates at peak and at trough.

Year	Percentage Ratio $\frac{100 \cdot \text{actual}}{\text{SL B}}$	Deviation %	Deviation Squared
1871	87	-13	169
72	86	-14	196
73	80	-20	400
74	106	+ 6	36
75	136	+36	1296
76	122	+22	484
77	105	+ 5	25
78	92	- 8	64
79	81	-19	361
1880	86	-14	196
81	83	-17	289
82	84	-16	256
83	90	-10	100
84	125	+25	625
85	105	+ 5	25
86	79	-21	441
87	106	+ 6	36
88	95	- 5	25
89	103	+ 3	9
1890	97	- 3	9
91	108	+ 8	64
92	94	- 6	36
93	97	- 3	9
94	97	- 3	9
95	115	+15	225
96	117	+17	289
97	84	-16	256
98	98	- 2	4
99	103	+ 3	9
1900	92	- 8	64
01	97	- 3	9
02	106	+ 6	36
03	94	- 6	36
1904	90	-10	100
1905	115	+15	225
06	106	+ 6	36
07	98	- 2	4
08	89	-11	121
09	98	- 2	4
1910	107	+ 7	49
11	107	+ 7	49
12	108	+ 8	64
13	97	- 3	9
14	95	- 5	25
15	91	- 9	81
16	113	+13	169
17	102	+ 2	4
18	78	-22	484
19	98	- 2	4
1920	105	+ 5	25
21	93	- 7	49
22	99	- 1	1
23	102	+ 2	4
24	96	- 4	16
25	108	+ 8	64
26	106	+ 6	36
27	96	- 4	16
28	95	- 5	25
29	94	- 6	36
1930	96	- 4	16
31	112	+12	144
32	110	+10	100
33	87	-13	169
34	74	-26	676
35	93	- 7	49
36	111	+11	121
1937	109	+ 9	81

$\Sigma(d^2) = 8924$

$$sd = \sqrt{\frac{8924}{67}} = 11.5\%$$

Table Ea is concluded on next page

Table Ea (concluded) San Francisco Real Estate Activity

Time Lengths of the Phases or Quarters of the Cycle of Actual Annual Figures about SL B - the Short Business Cycle. Read from Chart 2.

Cycle Number	Lengths in Years rp	pf	ft	tr
1	1.4 yr	0.8 yr	3.0 yrs	1.5 yr
2	0.9	1.7	3.5	2.9
3	0.9	0.9	0.9	0.7
4	0.5	0.5	0.5	0.5
5	0.4	0.3	0.6	0.5
6	0.4	0.7	1.6	1.1
7	1.4	0.8	0.5	1.0
8	0.7	0.5	0.7	1.1
9	0.9	0.6	1.2	0.4
10	0.9	2.0	1.0	0.9
11	1.8	2.1	1.8	0.8
12	0.8	0.9	0.7	1.2
13	0.9	0.5	0.4	1.0
14	1.1	0.5	0.4	0.1
15	0.9	1.6	2.2	1.3
16	1.1	0.8	1.7	1.7
17	1.1	0.9	0.7	1.5
Average Lengths	0.9 yr	0.9 yr	1.3 yr	1.1 yr

Total length of a typical cycle 4.2 yrs.

Percentage Deviations of Actual from SL B at Peaks and Troughs

At Peak Year	Deviation
1875	36%
84	25
87	6
89	3
91	8
96	17
99	3
1902	6
05	15
11	7
16	13
20	5
23	2
25	8
31	12
1937	9
Average Deviation	10.9% at peak

At Trough Year	Deviation
1873	20%
80	14
86	21
88	5
90	3
93	6
97	6
1900	8
04	8
08	11
15	9
18	22
21	7
24	4
29	6
1934	26
Average Deviation	10.9% at trough

Table Eb. SAN FRANCISCO REAL ESTATE ACTIVITY

Calculation of Standard Measures of the Major Cycle, from the Record 1867 to 1940;

the amplitude or standard deviation;

the typical lengths of the phases or quarters of the cycle;

the typical ordinates at peak and at trough.

Year	Percentage Ratio $\frac{100.\text{SL B}}{\text{SL M}}$	Deviation %	Deviation Squared
1883	75%	-25	625
84	77	-23	529
85	86	-14	196
86	98	- 2	4
87	116	+16	256
88	139	+39	1521
89	160	+60	3600
1890	167	+67	4489
91	150	+50	2500
92	125	+25	625
93	100	0	0
94	81	-19	361
95	69	-31	961
96	62	-38	1444
97	57	-43	1849
98	57	-43	1849
99	60	-40	1600
1900	70	-30	900
01	84	-16	256
02	102	+ 2	4
03	122	+22	484
04	139	+39	1521
05	138	+38	1444
06	136	+36	1296
07	130	+30	900
08	124	+24	576
09	117	+17	289
1910	110	+10	100
11	101	+ 1	1
12	92	- 8	64
13	84	-16	256
14	76	-24	576
15	68	-32	1024
16	63	-37	1369
17	62	-38	1444
18	63	-37	1369
19	72	-28	784
1920	85	-15	225
21	98	- 2	4
22	116	+16	256
23	131	+31	961
24	146	+46	2116
25	150	+50	2500
26	140	+40	1600
27	122	+22	484
28	107	+ 7	49
29	93	- 7	49
1930	84	-16	256

$\Sigma(d^2) = 45566$

$sd = \sqrt{\frac{45566}{48}} = 30.8\%$

Time Lengths of the Phases or Quarters of the Major Cycle - the cycle of SL B about SL M. Read from Chart 3.

Cycle Number	Lengths in Years ft	tr	rp	pf
1	8.9 yrs.	6.0 yrs.	3.2 yrs	2.5 yrs.
2	5.3	4.2	3.2	6.2
3	5.8	3.4	4.3	4.0
4	4.1	5.2		
Average Lengths	6.0 yrs.	4.7 yrs.	3.6 yrs.	4.2 yrs.

Total length of a typical cycle, 18.5 yrs.

Percentage Deviations of SL B from SL M at Peaks and Troughs

At Peak Year	Deviation	At Trough Year	Deviation
1890	67%	1897-98	43%
1904	39	1917	38
1925	50		
Average Deviation	51.7% at peak		40.5% at trough

In Table E(a), the standard measures of the short business cycle are calculated; they give a reasonably complete picture of the typical cycle of that order. To illustrate their use, that typical cycle is constructed, through several recurrences, on Chart 4; they are employed in more realistic fashion on Chart 5, to project the short business cycle into the future as a fluctuation about the curving projected line of SL B.

IN Table E(b), the standard measures of the major cycle are calculated. They are used similarly on Charts 4 and 5, to describe this element in the time series, and to forecast.

SAN FRANCISCO REAL ESTATE ACTIVITY

Comparison of the Two Orders of Fluctuation (Table E, parts a and b, and Chart 4)

It will be noted that for this real estate series the amplitude of the short business cycle is slight, as the standard deviation (sd) is only 11.5%, whereas the amplitude of the major cycle is three times as great. This presents a very significant finding to men engaged in the real estate business. The short business cycle has little effect upon their activity and their earnings, but in the major cycle, after these men have flourished during several years of boom activity, and the down-turn has come, it will likely be at least ten years before activity is resumed in real estate. During those years, their inactivity may be disastrous.

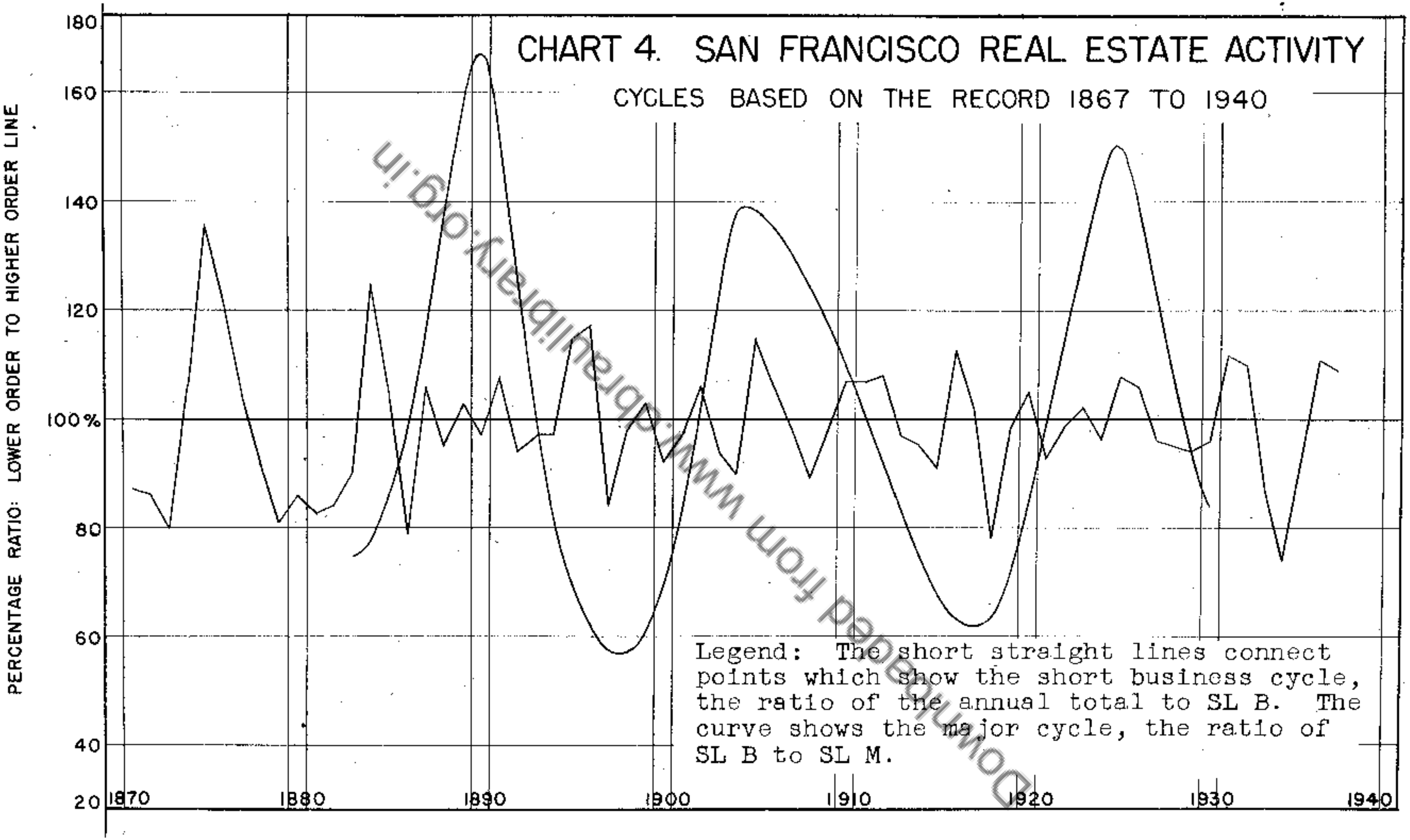


CHART 4. SAN FRANCISCO REAL ESTATE ACTIVITY

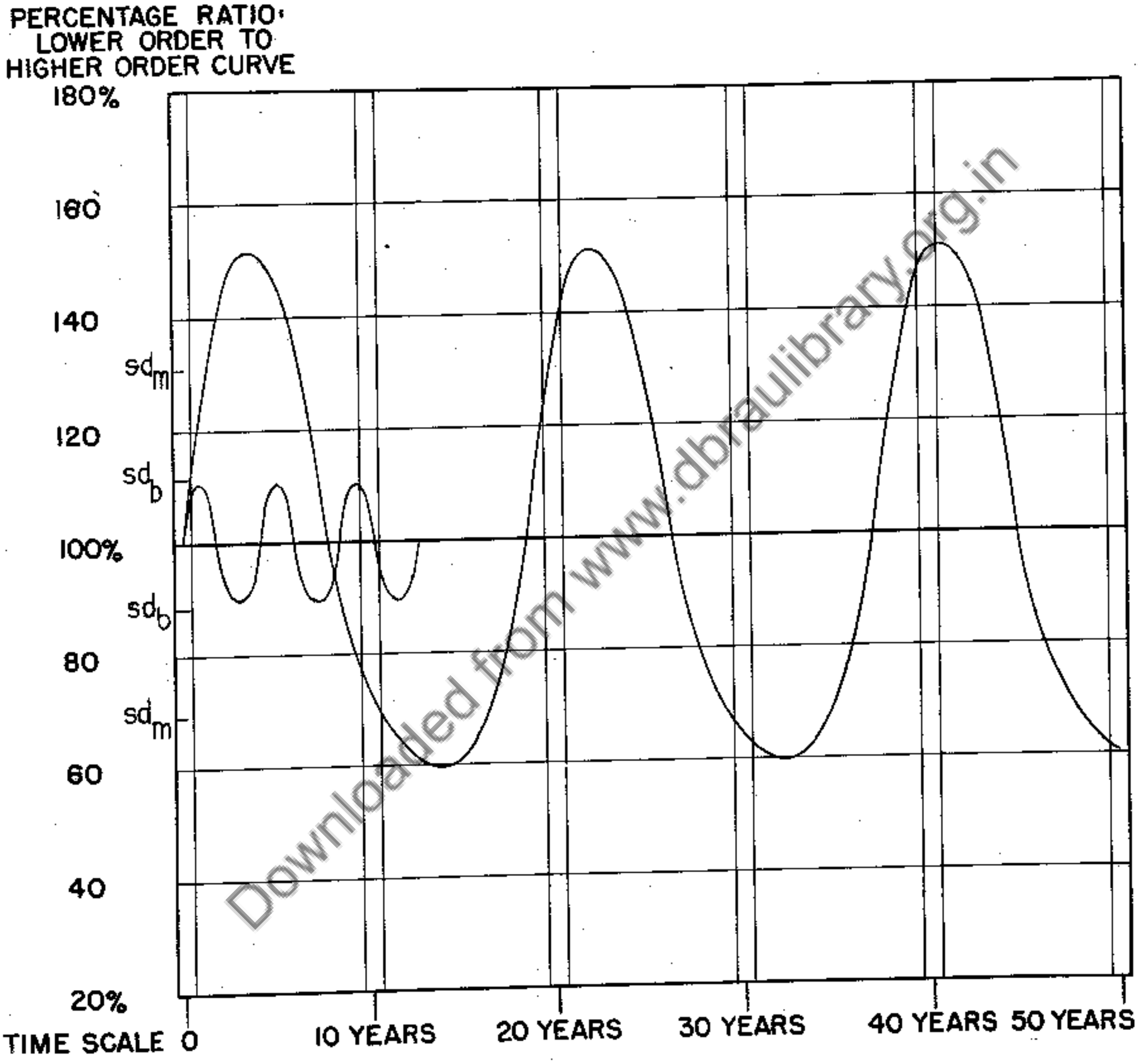
CYCLES BASED ON THE RECORD 1867 TO 1940

Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual total to SL B. The curve shows the major cycle, the ratio of SL B to SL M.

CHART 5.

SAN FRANCISCO REAL ESTATE ACTIVITY

Standard or Typical Cycles, Constructed from the Standard Measures of the two Orders of Cycles, as Calculated from the Record 1867 to 1940.



(from the date of the first "r" phase point)

Legend: In the left margin, the two marks above and below the 100% line, labeled sdm, show the amplitude of the major cycle, 30.8%. The marks labeled sdb show the amplitude of the short business cycle, 11.5%.

CHAPTER IV.

APPLICATION TO CORRELATION AND FORECASTING

Section 1. THE COMPARISON OF TIME SERIES: CORRELATION

FOR time series that have been broken into their elements by fitting a mathematical trend, calculating normal, and then measuring deviations, the procedure of comparing two or more series has been standardized. One calculates the ratios of actual to normal, and from these ratios the Pearsonian coefficient of correlation of the cyclical movements. This procedure has been challenged by critics who contend that time series do not afford the underlying independence of observations and random distribution of forces which are necessary conditions to the Pearsonian calculation of correlation. Whether or not one rejects the correlation calculation, it would seem reasonable to insist that the operator supply an extensive verbal comparison of the characteristics of the two series.

THIS verbal comparison can be expanded richly under the method of smoothing by stages. The comparison may now be organized to include the shape of the trend, the standard measures of each order of cycle, and all significant dates in the several smoothing lines.

IN addition to the verbal comparison, a numerical coefficient of correlation may be separately calculated for each order of cycle - for whatever these coefficients are worth. The significant novelty should be noted, that more than one coefficient can be secured. (Because of the fixed period, it would not be suitable to calculate the third correlation, that for the annual cycle, the seasonal movement).

THE short business cycles of the two series may be submitted to the Pearsonian calculation, to secure the coefficient of correlation, and lag. This relationship between the short business cycle movements should also be studied in a theoretical (not merely statistical) fashion. It may be concluded, for example, that, in the short movement, the size of the wheat crop has a causal relation to the price of wheat.

THE major cycle may be studied separately, the coefficient of correlation calculated, lag determined, and economic theory again enlisted. It may be concluded, for this long movement, that the price of wheat over a period of years has a causal relation to the acreage planted, and consequently to the size of the wheat crop, reversing the causal relationship in the short cycle.

Section 2. FORECASTING.

THE analysis of time series by smoothing by stages gives an organized set of measurements which make possible an improvement in the procedure of forecasting, from the accustomed forward projection of the normal line.

ASSUME that SL M is the final smoothing line you have been able to draw -- the nearest approach to the underlying secular trend. Proceed by these steps: (1) Continue the trend, SL M, forward, in its recent slope and curvature, as on Chart 6. (2) About that projected trend as a base, carry forward the typical shape of the major cycle, joining this sinuous forecasting line to SL B. Probably one full cycle is as far as one should presume to carry this portion of the forecast -- because values that lie many years in the future are highly uncertain and indeterminate. (3) About this second forecasting line, the projection of SL B, carry forward the typical shape of the next shorter cycle, the short business cycle, joining this third, and even more sinuous forecasting line to SL A, again proceeding no more than one full cycle -- likely about three or four years. (4) Superpose upon the forecasting line SL A, last drawn, the standard seasonal pattern, connecting this fourth and final forecasting line with the time polygon of the actual quarterly data. In utilizing the standard measures of this seasonal cycle, because of the constant period and the moderate uniformity in shape, one may dare to project several cycles into the future -- say the full distance to the end of forecasting line SL A. (This fourth step, the seasonal elaboration, is omitted on Chart 6).

EACH of the forecasting lines may be regarded as clearly defined at near dates, but as growing less distinct in vertical position or ordinate as it proceeds farther into the future. As one considers more and more distant future dates, he should give less attention to the more sinuous forecasting lines, those containing the seasonal and short-cycle movements, and more to the basic lines -- and finally, after 12 or 15 years, to the "trend," SL M, alone.

ON comparing forecasting by the method of successive smoothings, with the method of extending the normal line, it will be seen that in both methods one mechanically carries forward regular movements that have been discovered in the past record of the variable. But, as has been pointed out in the Preface, no mechanical procedure can give a complete and adequate forecast, for it cannot make allowance for an expectation of change from the old pattern. The mechanical projection of a curve or a set of curves into the future, should, therefore, be improved by making allowances for such expectation of change; the mechanically determined curves should be altered or bent to conform to a reasonable expectation of change. (This subject, the consideration of a departure from the mechanical forward projecting of a set of standard patterns, is discussed again below, in connection with Chart 6).

SAN FRANCISCO REAL ESTATE ACTIVITY; A FORECAST

Based on the record 1867 to 1940. (a discussion to accompany Chart 6)

THE record of San Francisco real estate activity from 1867 to 1940, has been studied from annual data, without a seasonal analysis. The forecast, below, has been prepared with the same omission of monthly detail; this is in order to save elaboration. But in any practical case, where business or government decision rests on the findings, it is likely that the more elaborate calculation will be made, and the seasonal element included in the forecast.

GIVEN the original time series; given the two smoothing lines SL B and SL M, from Table c and Charts 2 and 3; given the standard measures of the two orders of cycles, from Table E and Chart 5. Chart 6 is constructed as will be described (though without here presenting the full figures - the reader may care to fill in the work table that must accompany and direct this construction).

THE last thirty years of Charts 2 and 3 (1910 to 1940) were copied to Chart 6, that is, the original data, the final location of SL B, including the dotted end portion, and the final location of SL M.

SL M was projected into the future, with consideration to its recent slope and curvature, and yet with what seemed a reasonable forecast of real estate activity in the community. When this had been done, it was found that the steeply declining dotted end of SL M that had been copied from Chart 3, did not offer a suitable joining with the new forecasting line; consequently, the dotted portion of the line was rejected; it had been only tentative in the first instance.

THE following steps were taken in order to apply the typical form of the major cycle as a fluctuation about the extension of SL M, thereby to secure a forecasting extension of SL B: tentatively, the date of the intersection of the dotted end of SL B with the relocated extension of SL M was accepted as a date from which to draw the major cycle. This point was taken as an approximation to r_4 in the major cycle. The standard phase lengths of the major cycle are: rp 3.6 years, pf 4.2 years, ft 6.0 years, and tr 4.7 years; total length of the typical cycle 18.5 years. These distances were laid out from r_4. At the f and r dates, the values of SL M were taken to be the same as those of SL B, for these are points where the two curves are expected to intersect. At peak dates, the value of SL M was multiplied by 1.517, to determine the expected value of SL B; and at troughs, the value of SL M was multiplied by 0.595 (for the average deviation of SL B from SL M at the peak had been determined to be + 51.7%, and at the trough - 40.5%). But when tentative SL B had been sketched, from the approximation to r_4, through these phase points, it was found that the dotted end of SL B did not make a good joining with it. So the joining of the sinuous forecasting line with the solid portion of SL B was made to conform to the criterion of gentle curvature; this caused the rejection of the dotted end, which of course had been but tentative anyhow; and this gave the final location of the intersection r_4. From this new base point in time, the necessary corrections were made in laying out the time lengths to the future phase points; the values of SL M at these phase points were used to recalculate the values of SL B (equal values at points r and f; multiplied by the factors above indicated, at points p and t).

TO review what has been done so far, this forecasting extension of SL B is based upon:

(1) A somewhat subjective projection of SL M, which conforms to the general instruction that the forecasting trend line should project the trend into the future "at the recent slope and curvature".

(2) The formal application of the typical shape of the major cycle, as a fluctuation about the projection of SL M. Even the second guide, the standard form of the major cycle, might have been subjected to modification, on the basis of well-informed critical judgment -- again an introduction of the subjective element. The operator is under no necessity to accept without change the "typical" ordinates at p and at t. He may choose to question the representativeness, in the original time series, of the boom period of the 1920's. It followed the opening of the Panama Canal, the victorious end of the First World War, and a great westward migration; and it preceded the building of the two great San Francisco bridges, to Oakland and to Marin County, which in the future may move real estate activity away from San Francisco, the urban center proper. Probably, future booms will not be so extreme as that of the 1920's, within the present limits of San Francisco. But no such subjective modification of the major cycle was made on Chart 6; the "typical" form of the major cycle was used.

(3) The final step in the forecast was to draw the typical fluctuation of the annual data about the forecasting extention of SL B. For this purpose, the date of f_{17} was accepted from Chart 2 and Table C as the base from which to measure, and the cycles and phases of the future short business cycles were measured from that base. The standard phase lengths of the short business cycle are: rp, 0.9 year; pf, 0.9; ft, 1.3; tr, 1.1, total length of the typical cycle, 4.2 years. The typical ratio of actual to to SL B at the peak is 110.9 per cent; at the trough,89.1 per cent.

THE "expected" annual count of deeds for the trough in 1939 was not plotted; nor was r_{18}, in 1940, made use of on the chart. Instead, the latest available actual count, that for 1940, was connected to the next projected or forecasting phase point, p_{18}, in 1941, and the forecasting line was carried forward to the succeeding forecasting phase points. A curved line was used instead of straight lines for connecting the future phase points (straight lines would not be helpful, as the phase points do not fall precisely at mid-year dates, which would represent calendar years). This curved line may be thought of as a forecasting extension of SL A, because it substitutes a curve for the time polygon which would connect discrete annual figures.

THE forecasting system of lines is not complete, because the seasonal elaboration has been omitted. For such application or completion, the values of the forecasting extension of SL A would be read from Chart 6 at monthly or quarterly intervals, and would be multiplied by the seasonal indexes.

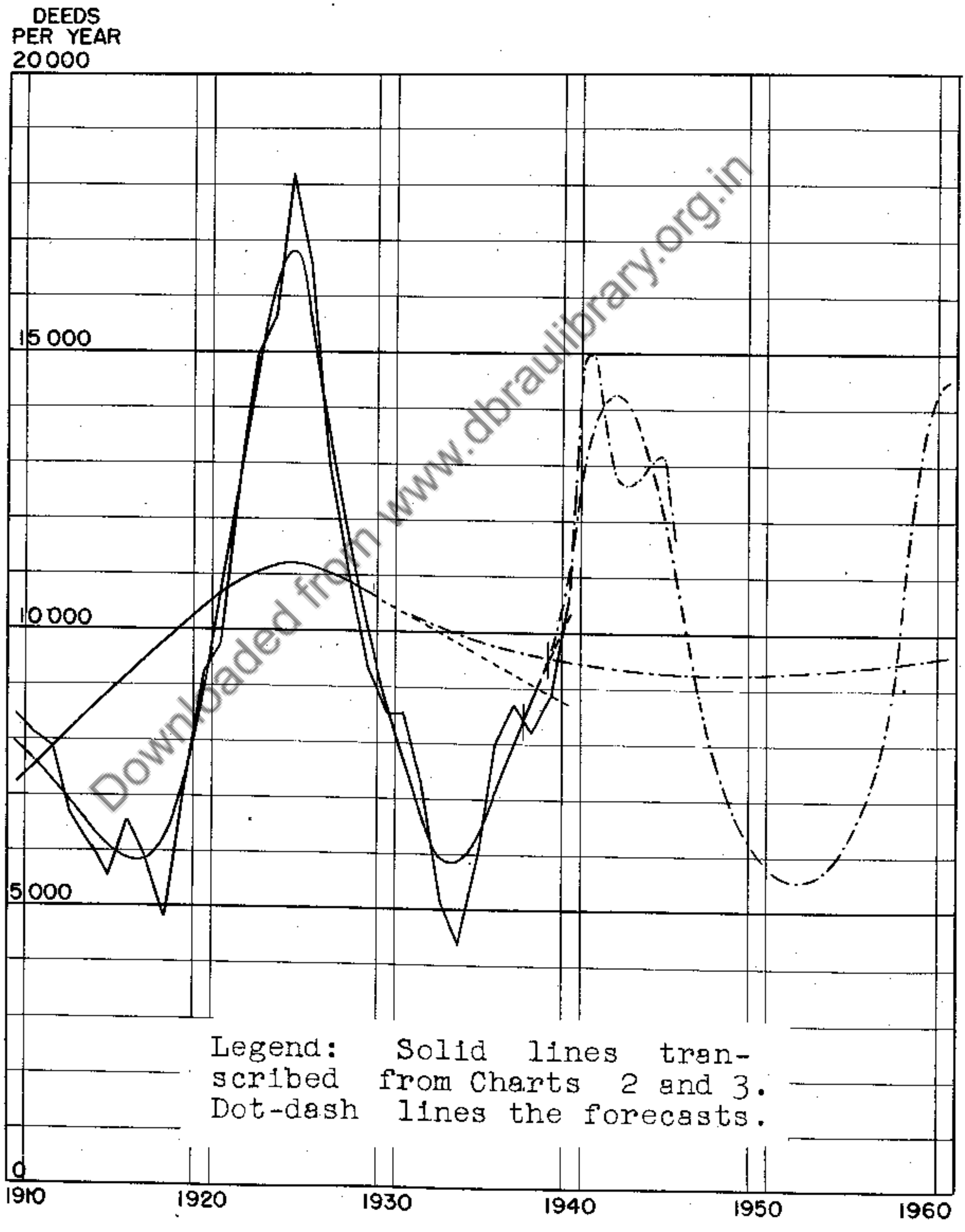
THE whole forecasting system, the set of forecasting lines, is largely mechanical, though the discussion has showed that personal judgment has been used in drawing SL M, and might well have been used again in altering the "typical" form of the major cycle (in drawing SL B to fluctuate about SL M). Personal judgment might even lead one to modify the standard measures of the short business cycle (in drawing SL A).

THE whole forecast in the case of this real estate series or of any other variable must be in some doubt, because of the probable lack of homogeneity of forces acting in the future with those acting in the past. One cannot expect the future to be like the past in such matters as war and peace, as the zoning of city lands, tax rates, the loan policies of federal and other agencies, such matters as the new (and financially irresponsible?) class of home owners that have been tempted into the field by recent federal policies of low interest rates and small down payments, and such as foreign trade practices, which are so important to this port city.

CHART 6.

SAN FRANCISCO REAL ESTATE ACTIVITY

A FORECAST TO 1960, BASED ON THE RECORD 1867 TO 1940



Legend: Solid lines transcribed from Charts 2 and 3. Dot-dash lines the forecasts.

CHAPTER V.

ANALYSIS OF SEVEN SERIES

TO demonstrate the method of smoothing by stages, seven series will now be examined; full tables of calculations and work charts will be presented. These series have been selected from a number examined by Simon S. Kuznets in Secular Movements in Production and Prices. (1)

IN each of the seven analyses, SL B and SL M are located, with correction for curvature; and the standard measures of the two orders of cycles are calculated. Then, to facilitate direct comparison with Kuznets, one of my standard measures, the amplitude or standard deviation, is recalculated for the same limited period as his figures cover. This comparison cannot be made perfect, however, for in the method of smoothing by stages it is the practice to omit the first and last half-cycle from the calculation of the amplitude, whereas Kuznets' figures for the full period are used.

KUZNETS began his analysis of each series by fitting a trend - a logistic curve for each of the six quantity series, and a parabola for the price series; the values of the trend he entered in the second column of the table in his book. The ratio of actual to trend, he entered in column III: this we shall call the composite cycle. He smoothed it by moving averages salted with subjective judgment, thus gaining a representation of the major cycle; he entered the value in column IV. In column V, he entered the ratio of column III to column IV; this is his short business cycle. The present writer divided column I (actual annual values) by column V, to secure what he takes the liberty of calling "Kuznets' intermediate trend," an equivalent of Smoothing Line B.

KUZNETS' figures are not here reproduced. The five columns may be found in the tables in his book, and the sixth set of figures may be recalculated in a few moments by simple division. But the results are here presented in charts, the equations of the trend lines are reported, and the amplitudes of the two orders of cycles (as calculated from Kuznets' figures by the present writer). Some textual comparisons and notes will also be found.

IT will be seen that in the present study, the seven series have been brought to a more recent date than 1925, which is the last year reported by Kuznets. It is hoped that no unjust comparison has been made with Kuznets' results merely on this basis of later information.

IN general, the method of smoothing by stages gives a smaller value for the standard deviation or amplitude of each order of cycle than does Kuznets' method; this statement is particularly true for the major cycle. But it is not clear how much credit should be claimed for the new method merely because it excels in the closeness of the fit of the trend line. The superiority in closeness of fit is to be expected from an empirical method which from the beginning sets out to follow the data. However, the fact of the smaller value of the standard deviation is of sufficient interest to warrant reporting.

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(1) Boston, Houghton Mifflin, 1930.

FOR each series, not only may the trends be seen on the charts, but also the shape of the trend secured by Kuznets is verbally compared with the shape of SL M secured by the method of smoothing by stages. The reader may care to refer back to the Preface, to the discussion of the underlying forces affecting time series, and of the appropriate shapes and types of trends.

KUZNETS was testing his hypotnesis that industrial production and agricultural output can best be fitted by an S-shaped curve of the logistic type. The present study, in which trend lines are fitted by a wholly empirical method, substantiates Kuznets' thesis; for the smoothing lines so secured agree remarkably closely with his logistic trend lines. The agreement does not extend to the price series, however.

THE reader will observe that there is not complete uniformity, under the method of smoothing by stages, in handling certain small matters of procedure. In one instance, the location of SL B seems so obvious that it is drawn without the objective check of moving averages. The correction for curvature is handled slightly differently in several cases. Some of the series are checked by an arithmetic type of moving average, and others by a geometric. These are offered as permissible variations within the system.

Section 1. WHEAT PRICES.

A comparison of Kuznets' results with those obtained by the method of smoothing by stages:

Kuznets' trend, a parabola, has the equation:

$y = 105.60 - 10.214x + 0.809x^2$. The origin is taken in the year 1863, and x is measured in units of five years. The equation gives the values, calculated by Kuznets, entered in the table below. Opposite each of these values is given also the value of Smoothing Line M as calculated by the method of smoothing by stages.

	Kuznets' Trend	SL M
1868	$ .962	$1.022
1873	.884	1.000
1878	.822	.953
1883	.777	.855
1888	.748	.745
1893	.734	.690
1898	.748	.693
1903	.767	.745
1908	.802	.840
1913	.854	1.115
1915	.876	1.300

SMOOTHING Line M fits closely to the data, as is evidenced by the low standard deviation in the major cycle for the period 1866 to 1915. This price series places Kuznets at more of a disadvantage than do the volume series which follow. A parabola makes a most unsatisfactory trend line. It cannot safely be extended at either end, and must be regarded as no more than a smoothing device. No logical explanation of the changes in the value of the variable can be based upon a parabola. If one must fit a trend to a curve with a sharp dip in it, he might better use a skewed distribution curve, or one of the periodic curves, for example a modified sine curve.

AS has been contended in the Preface, any "total" mathematical trend fitted to a price series rests upon a false assumption of homogeneity of the underlying forces My Smoothing Line **M** cannot be considered the ultimate trend line, the so-called secular trend. But neither can Kuznets' parabola. Suppose the data covered several hundred years and included a number of periods of war-inflation. Under such circumstances, Kuznets' present parabola would take infinite values; SL M would rise and fall with each such inflationary movement; it would itself exhibit a number of cycles. Through those cycles, or long waves, in SL M, another smoothing line could be drawn which would be one step nearer to the "secular" trend, the trend through the centuries.

THE major cycles secured by the two methods look reasonably similar prior to 1905, but after that date the resemblance ceases, as Smoothing Line **M** moves into a definitely higher level than does Kuznets' trend.

A question may be raised as to the limitation by Kuznets of the data in this series to the year 1915, although his other series were carried on to 1925. Was this done to secure a homogeneous period, free from the inflationary effects upon prices during the first World War? The period ending in 1915 succeeds in escaping war-time inflation of prices.The contrast which the standard deviation (below) for that short period, offers to the standard deviation for the full record, shows the effect of the inflation, as this measure of amplitude is practically doubled in the longer period. In fact, the great violence of price movements during an inflation-deflation episode raises the question whether they belong in the same "statistical universe" with price movements during the economic stability of peace-time. Possibly Kuznets was right in stopping with 1915. (See reference to Silberling in the Preface.)

WHEAT PRICES.

The values of the standard deviation, by the several calculations, may be shown in tabular form:

	From Kuznets' figures, based on period 1866 to 1915	Figures secured by the method of smoothing by stages	
		based on period 1866 to 1915	based on period 1866 to 1938
The short business cycle	14.1%	12.2%	11.9%
Years included (omit terminal half-cycles)	full	1871 to 1914	1871 to 1935
The major cycle	10.2%	8.6%	15.3%
Years included (omit terminal half-cycles)	full	1883 to 1910	1883 to 1930

Table F. UNITED STATES DECEMBER FARM PRICES OF WHEAT, 1866 TO 1938

Two stages of smoothing. Columns numbered as in Tables B and C.

Source: Yearbooks of the United States Department of Agriculture.

Part a (four pages)

1 Year	2 (or 4) Wheat Price (cents per bushel)	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle	8	9	10 Length in Years	11a Moving Total	11b Moving Cyclical Average ¢/bu.	12 Smoothing Line B ¢/bu.	13 Phase Point (major cycle)	20 Second Approx. to Smoothing Line M ¢/bu.	24 Final Smoothing Line M ¢/bu.	1 Year
				Begin	Middle	End								
1866	108.4									(87.0)			(106.2)	1866
67	105.1	f_1								(86.5)			(105.9)	67
68	77.1									(86.5)			(105.5)	68
69	57.5	t_1								(86.0)			(105.1)	69
1870	82.1		$f_{1,2}$	1868	1870	1873	6	512.9	85.4	(86.5)			(104.8)	1870
71	102.5	r_1								87.0			(104.0)	71
72	99.1	p_1	$t_{1,2}$	1869	1872	1874	6	512.8	85.4	87.0			(103.3)	72
73	94.0		$r_{1,2}$	1871	1873	1876	6	538.0	89.6	87.5			(102.7)	73
74	77.6	f_2	$p_{1,2}$	1872	1874	1877	6	536.4	89.4	88.0			(101.9)	74
75	77.9	t_2								89.0			(101.0)	75
76	86.9	r_2	$f_{2,3}$ $t_{2,3}$	1874 1875	1875-76 1876	1877 1878	4 4	343.3 342.2	85.8 85.6	90.0			(100.0)	76
77	100.9	p_2	$r_{2,3}$	1876	1877	1878	3	264.3	88.1	91.5			(98.9)	77
78	76.5	f_3 t_3	$p_{2,3}$	1877	1878	1879	3	280.0	96.0	94.0	r_1	95.3	(97.7)	78
79	110.6	r_3 p_3	$f_{3,4}$ $t_{3,4}$	1878 1878	1878-79 1879	1879 1880	2 3	187.1 282.2	93.6 94.0	99.0		93.8	(96.0)	79
1880	95.1	f_4 t_4	$r_{3,4}$ $p_{3,4}$	1879 1879	1879-80 1880	1880 1881	2 3	205.7 324.9	102.8 108.3	106.5		92.0	(94.4)	1880
81	119.2	r_4 p_4	$f_{4,5}$ $t_{4,5}$	1880 1880	1880-81 1881	1881 1882	2 3	214.3 302.7	107.2 100.9	106.0	P1	90.0	(92.5)	81

Table F, Part a (continued) United States December Farm Prices of Wheat

1 Year	2 (or 4) Wheat Price (cents per bushel)	5 Phase Point (short cycle)	12 Smoothing Line B ¢/bu.	13 Phase Point (major cycle)	20 2nd Approx to SL M ¢/bu.	24 Final SL M ¢/bu.	1 Year
1882	88.4	f_5 t_5 r_5	98.0		88.0	(90.3)	1882
83	91.1	p_5 f_6	82.5	f_1	85.0	87.4	83
84	64.5	t_6 r_6	74.5		83.3	84.8	84
85	77.1	p_6	70.0	t_1	81.0	81.5	85
86	68.7	f_7	71.5		78.8	78.2	86
87	68.1	t_7	75.0	r_2	76.5	75.7	87
88	92.6	r_7 p_7	77.5		74.5	73.0	88
89	69.5	f_8 t_8	79.0	p_2	72.5	71.0	89
1890	83.3	r_8	77.5		71.0	69.3	1890
91	83.4	p_8	73.0		70.0	68.0	91
92	62.2	f_9	66.5	f_2	69.6	67.2	92
93	53.5		62.0		69.0	66.5	93
94	48.9	t_9	61.0		68.8	66.2	94
95	50.3	r_9	61.0		68.3	66.2	95
96	71.7		61.5	t_2	68.5	66.6	96
97	80.9	p_9	62.5		69.0	66.8	97
98	58.2	f_{10}	63.5		69.3	67.2	98
99	58.6	t_{10} r_{10}	64.0		70.0	67.9	99
1900	62.0	p_{10}	65.0		71.0	69.0	1900
01	62.6	f_{11}	66.5		72.0	70.0	01

6 Cycle	7 Yearly Figures Included in the Cycle: Begin	8 Middle	9 End	10 Length in Years	11a Moving Total	11b Moving Cyclical Average ¢/bu.
$r_{4,5}$	1881	1881-82	1882	2	207.6	103.8
$p_{4,5}$	1881	1882	1883	3	298.7	99.6
$f_{5,6}$	1882	1882-83	1883	2	179.5	89.8
$t_{5,6}$	1882	1883	1884	3	244.0	81.3
$r_{5,6}$	1883	1883-84	1884	2	155.6	77.8
$p_{5,6}$	1883	1884	1885	3	232.7	77.6
$f_{6,7}$	1884	1885	1886	3	210.3	70.1
$t_{6,7}$	1884	1885	1887	4	278.4	69.6
$r_{6,7}$	1885	1886	1887	3	213.9	71.3
$p_{6,7}$	1885	1886-87	1888	4	306.5	76.6
$f_{7,8}$	1886	1887	1888	3	229.4	76.4
$t_{7,8}$	1887	1888	1889	3	230.2	76.7
$r_{7,8}$	1888	1888	1889	2	162.1	81.0
$p_{7,8}$	1888	1889	1890	3	245.4	81.8
$f_{8,9}$	1889	1890	1892	4	298.4	74.6
$t_{8,9}$	1889	1891-92	1894	6	400.8	66.8
$r_{8,9}$	1890	1892	1895	6	381.6	63.6
$p_{8,9}$	1891	1894	1896	6	370.0	61.6
$f_{9,10}$	1892	1895	1898	7	425.7	60.8
$t_{9,10}$	1894	1896	1898	5	310.0	62.0
$r_{9,10}$	1896	1897-98	1899	4	296.4	67.4
$p_{9,10}$	1897	1898-99	1900	4	259.7	64.9
$f_{9,10}$	1898	1899-00	1901	4	241.4	60.4
$t_{10,11}$	1899	1900	1902	4	246.2	61.6
$r_{10,11}$	1900	1901	1903	4	257.1	64.2

(Table F, part a, is continued on next page)

Table F, Part a (continued) Wheat Prices, two stages of smoothing

1 Year	2 (or 4) Wheat Price (cents per bushel)	5 Phase Point	6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Middle	9 End	10 Length in Years	11a Moving Total	11b mca ¢/bu.	12 SL B ¢/bu.	13 Phase Point	20 2nd approx. to SL M ¢/bu.	24 Final SL M ¢/bu.	1 Year
1902	63.0	t_{11}	$p_{10,11}$	1900	1902	1904	5	349.5	69.9	69.0		73.0	71.3	1902
03	69.5	r_{11}	$f_{11,12}$	1901	1903	1905	5	362.1	72.4	72.5		74.5	73.1	03
04	92.4	p_{11}	$t_{11,12}$	1902	1904	1906	5	365.7	73.1	75.5	r_3	76.0	75.0	04
05	74.6	f_{12}	$r_{11,12}$	1903	1905	1907	5	389.2	77.8	79.0		77.5	76.6	05
06	66.2	t_{12}	$p_{11,12}$	1904	1906	1909	6	510.3	85.0	82.5		79.0	78.5	06
07	86.5	r_{12}	$f_{12,13}$	1905	1907	1909	5	417.9	83.6	86.0		81.5	81.5	07
08	92.2		$t_{12,13}$	1906	1908	1910	5	431.6	86.3	89.5		84.0	84.5	08
09	98.4	p_{12}	$r_{12,13}$	1907	1909	1910	4	365.4	91.4	92.0	p_3	88.0	87.8	09
1910	88.3	f_{13} t_{13} r_{13}	$p_{12,13}$ $f_{13,14}$ $t_{13,14}$	1909 1910 1910	1910 1910-11 1911	1911 1911 1912	3 2 3	274.1 175.7 261.7	91.4 87.6 87.2	91.0	f_3	92.0	92.4	1910
11	87.4	p_{13} f_{14}								85.3		98.0	97.5	11
12	76.0	t_{14}	$r_{13,14}$ $p_{13,14}$ $f_{14,15}$	1911 1911 1912	1912 1912-13 1913	1913 1914 1914	3 4 3	243.2 341.8 254.4	81.0 85.4 84.8	83.0		104.0	104.0	12
13	79.9	r_{14}								84.5	t_3	111.5	112.0	13
14	98.6	p_{14} f_{15}	$t_{14,15}$ $r_{14,15}$ $p_{14,15}$	1912 1914 1914	1914 1914-15 1915	1915 1915 1916	4 2 3	346.3 190.4 350.7	86.6 95.2 118.9	90.5		121.0	122.0	14
15	91.9	t_{15}								105.0		130.0	132.5	15
16	160.3	r_{15} p_{15}	$f_{15,16}$ $t_{15,16}$ $r_{15,16}$	1915 1915 1916	1916 1916-17 1917	1917 1918 1918	3 4 3	452.9 657.1 565.2	151.0 164.2 188.4	148.0	r_4	137.0	143.0	16
17	200.8									189.0		143.0	150.0	17
1918	204.2	f_{16} t_{16}	$p_{15,16}$	1917	1918	1919	3	621.2	207.0	206.5		147.5	156.0	1918

Table F, Part a (concluded) Wheat Prices, two stages of smoothing

1 Year	2 (or 4) Wheat Price ¢/bu.	5 Phase Point	6 Cycle	7 Yearly Figures Included in the Cycle	8	9	10 Length in Years	11a Moving Total	11b mca ¢/bu.	12 SL B ¢/bu.	13 Phase Point	20 2nd approx. to SL M ¢/bu.	24 Final SL M ¢/bu.	1 Year
				Begin	Middle	End								
1919	216.3	r_{16} p_{16}	$f_{16,17}$ $t_{16,17}$ $r_{16,17}$ $p_{16,17}$ $f_{17,18}$	1918 1918 1919 1919 1921	1919 1919-20 1920 1920-21 1921-22	1920 1921 1921 1922 1922	3 4 3 4 2	603.0 706.1 501.9 598.5 199.6	201.0 176.5 167.3 149.6 99.8	205.5	p_4	149.5	158.0	1919
1920	182.6									163.0		147.0	156.5	1920
21	103.0	f_{17} t_{17}								107.0	f_4	142.0	149.0	21
22	96.6	r_{17} p_{17}	$t_{17,18}$	1921	1922	1923	3	292.2	97.4	95.5	t_4	135.0	135.5	22
23	92.6	f_{18} t_{18}	$r_{17,18}$ $p_{17,18}$ $f_{18,19}$ $t_{18,19}$ $r_{18,19}$	1922 1922 1923 1923 1924	1923 1923-24 1924 1924-25 1925	1924 1925 1925 1926 1926	3 4 3 4 3	313.9 457.6 361.0 482.7 390.1	104.6 114.4 120.3 120.7 130.0	102.0		127.0	124.0	23
24	124.7	r_{18}								123.0	r_5	118.0	114.0	24
25	143.7	p_{18}								130.0	p_5	110.0	106.6	25
26	121.7	f_{19} t_{19}	$p_{18,19}$ $f_{19,20}$ $t_{19,20}$ $r_{19,20}$ $p_{19,20}$	1925 1926 1926 1927 1927	1926 1927 1927 1927-28 1928	1927 1927 1928 1928 1929	3 2 3 2 3	384.3 240.6 340.4 218.7 322.1	128.1 120.3 113.4 109.4 107.4	126.5		104.0	101.0	26
27	119.0	r_{19} p_{19}								114.0		99.0	96.3	27
28	99.8	f_{20} t_{20}								104.0		93.5	92.3	28
29	103.4	r_{20} p_{20}	$f_{20,21}$	1928	1929	1930	3	270.1	90.0	91.0	f_5	89.0	88.5	29
1930	67.0	f_{21}	$t_{20,21}$ $r_{20,21}$ $p_{20,21}$	1928 1929 1930	1930 1930-31 1931	1931 1932 1933	4 4 4	309.1 249.0 219.7	77.2 62.2 54.7	72.0		85.0	85.4	1930
31	39.0									53.0	t_5	82.5	(82.1)	31
32	37.9	t_{21}	$f_{21,22}$	1930	1932	1934	5	304.5	60.9	52.0		79.0	(79.5)	32
33	74.1	r_{21}	$t_{21,22}$	1932	1933	1935	4	281.7	70.4	65.0		76.5	(76.6)	33
34	84.8	p_{21}	$r_{21,22}$	1933	1934	1935	3	243.8	81.2	81.0	r_6	74.0	(74.2)	34
35	83.2	f_{22} t_{22} r_{22}	$p_{21,22}$	1934	1935	1936	3	272.3	90.8	91.0		71.5	(72.0)	35
36	102.6		$f_{22,23}$	1935	1936	1937	3	283.8	94.6	(94.5)	p_6	69.6	(69.8)	36
37	96.3	p_{22}								(91.0)			(67.5)	37
1938	56.1	f_{23}								(82.5)			(65.6)	1938

Table F. WHEAT PRICES

Part b part of the calculations for the second stage of smoothing, including correction for curvature.

Columns numbered as in Tables B and C.

14 Cycle	15 Yearly Figures Included in the Cycle Begin	16 Middle	17 End	18 Length in Years	19a Moving Total of SL B	19b Moving Cyclical Average (first application, ¢/bu.)	20 (to Table Fa) Second Approximation to SL M	21a Moving Total (of values of second approximation)	21b Reiterated mca, ¢/bu.	22a Deviation of Reiterated mca from first mca	22b from second approx. to SL M	22c Correction to first mca ¢/bu.	23 Adjusted mca ¢/bu.	24 (to Table Fa) Final SL M	16 Year
$r_{1,2}$	1878	1882-83	1887	10	877.5	87.7		864.2	86.4	- 1.3	- 1.6	+ 1.4	89.1		1882-83
$p_{1,2}$	1881	1885	1889	9	734.0	81.6		730.1	81.1	- .5	+ .1	0	81.6		1885
$f_{1,2}$	1883	1887	1891	9	680.5	75.6		693.1	77.0	+ 1.4	+ .5	- 1.0	74.6		1887
$t_{1,2}$	1886	1890-91	1896	11	765.5	69.6		787.5	71.6	+ 2.0	+ 1.1	- 1.6	68.0		1890-91
$r_{2,3}$	1888	1896	1904	17	1157.5	68.1		1207.0	71.0	+ 2.9	+ 2.5	- 2.7	65.4		1896
$p_{2,3}$	1890	1899	1908	19	1338.0	70.5		1382.0	72.7	+ 2.2	+ 2.7	- 2.5	68.0		1899
$f_{2,3}$	1892	1901	1910	19	1370.5	72.1		1421.0	74.8	+ 2.7	+ 2.8	- 2.7	69.4		1901
$t_{2,3}$	1896	1904-05	1913	18	1373.0	76.3		1458.8	81.0	+ 4.7	+ 4.0	- 4.3	72.0		1904-05
$r_{3,4}$	1904	1909	1915	12	1044.0	87.0		1142.5	95.2	+ 8.2	+ 7.2	- 7.7	79.3		1909
$p_{3,4}$	1909	1913-14	1918	10	1175.0	117.5		1172.0	117.2	- .3	+ 1.0	0	117.5		1913-14
$f_{3,4}$	1910	1915-16	1920	11	1451.5	132.0		1380.5	125.5	- 6.5	- 8.0	+ 7.2	139.2		1915-16
$t_{3,4}$	1913	1917-18	1922	10	1394.5	139.5		1363.5	136.4	- 3.1	- 8.8	+ 5.9	145.4		1917-18
$r_{4,5}$	1916	1919-20	1924	9	1339.5	148.8		1246.0	138.4	-10.4	-10.2	+10.3	159.1		1919-20
$p_{4,5}$	1919	1922	1925	7	926.0	132.2		928.5	132.6	+ .4	- 2.4	+ 1.0	133.2		1922
$f_{4,5}$	1921	1925	1929	9	993.0	110.3		1017.5	113.1	+ 2.8	+ 3.1	- 2.9	107.4		1925
$t_{4,5}$	1922	1927	1931	10	1011.0	101.1		1043.0	104.3	+ 3.2	+ 5.3	- 4.2	96.9		1927
$r_{5,6}$	1925	1929	1934	10	888.5	88.8		892.5	89.2	+ .4	+ .2	0	88.8		1929
$p_{5,6}$	1926	1931	1936	11	944.5	85.9		923.6	84.0	- 1.9	+ 1.5	0	85.9		1931

Note. On the worksheet, the cycles did not follow, as here, on consecutive lines; each was entered in the vertical position corresponding to its mid-date, column 16.

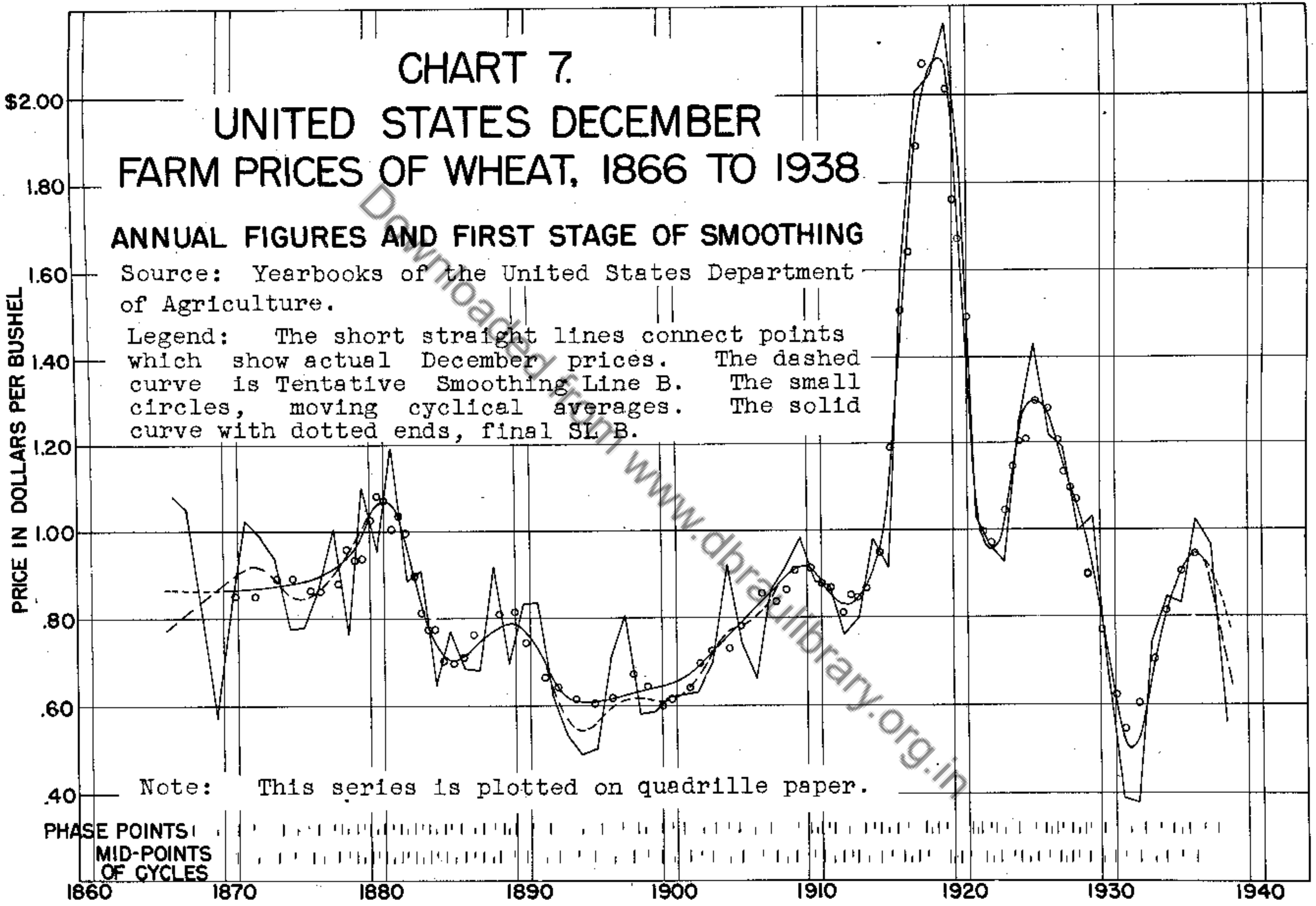


CHART 7.

UNITED STATES DECEMBER FARM PRICES OF WHEAT, 1866 TO 1938

ANNUAL FIGURES AND FIRST STAGE OF SMOOTHING

Source: Yearbooks of the United States Department of Agriculture.

Legend: The short straight lines connect points which show actual December prices. The dashed curve is Tentative Smoothing Line B. The small circles, moving cyclical averages. The solid curve with dotted ends, final SL B.

Note: This series is plotted on quadrille paper.

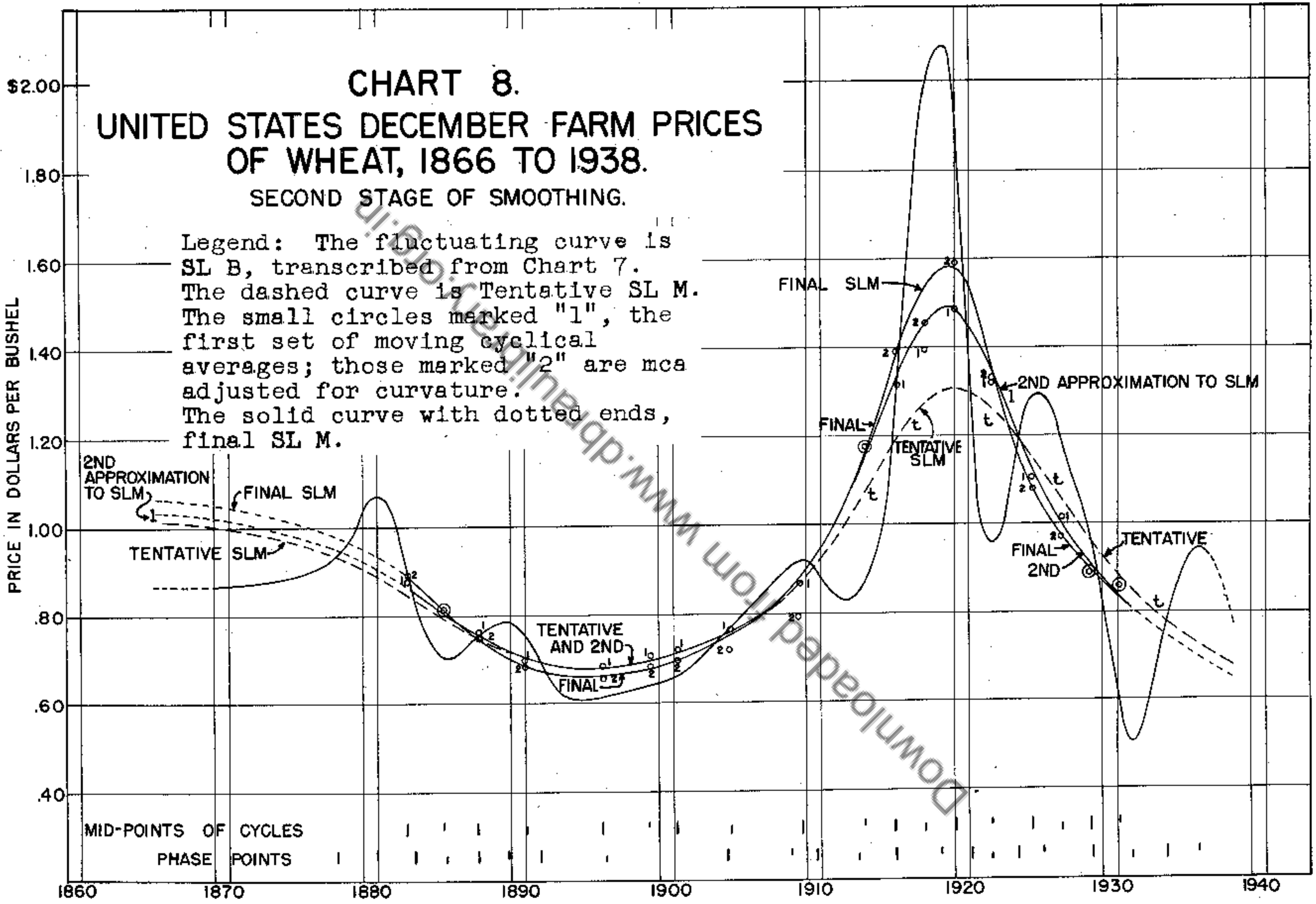


CHART 8.

UNITED STATES DECEMBER FARM PRICES OF WHEAT, 1866 TO 1938.

SECOND STAGE OF SMOOTHING.

Legend: The fluctuating curve is SL B, transcribed from Chart 7. The dashed curve is Tentative SL M. The small circles marked "1", the first set of moving cyclical averages; those marked "2" are mca adjusted for curvature. The solid curve with dotted ends, final SL M.

CHART 9.

UNITED STATES DECEMBER FARM PRICES OF WHEAT.

CYCLES BASED ON THE RECORD 1866 TO 1938

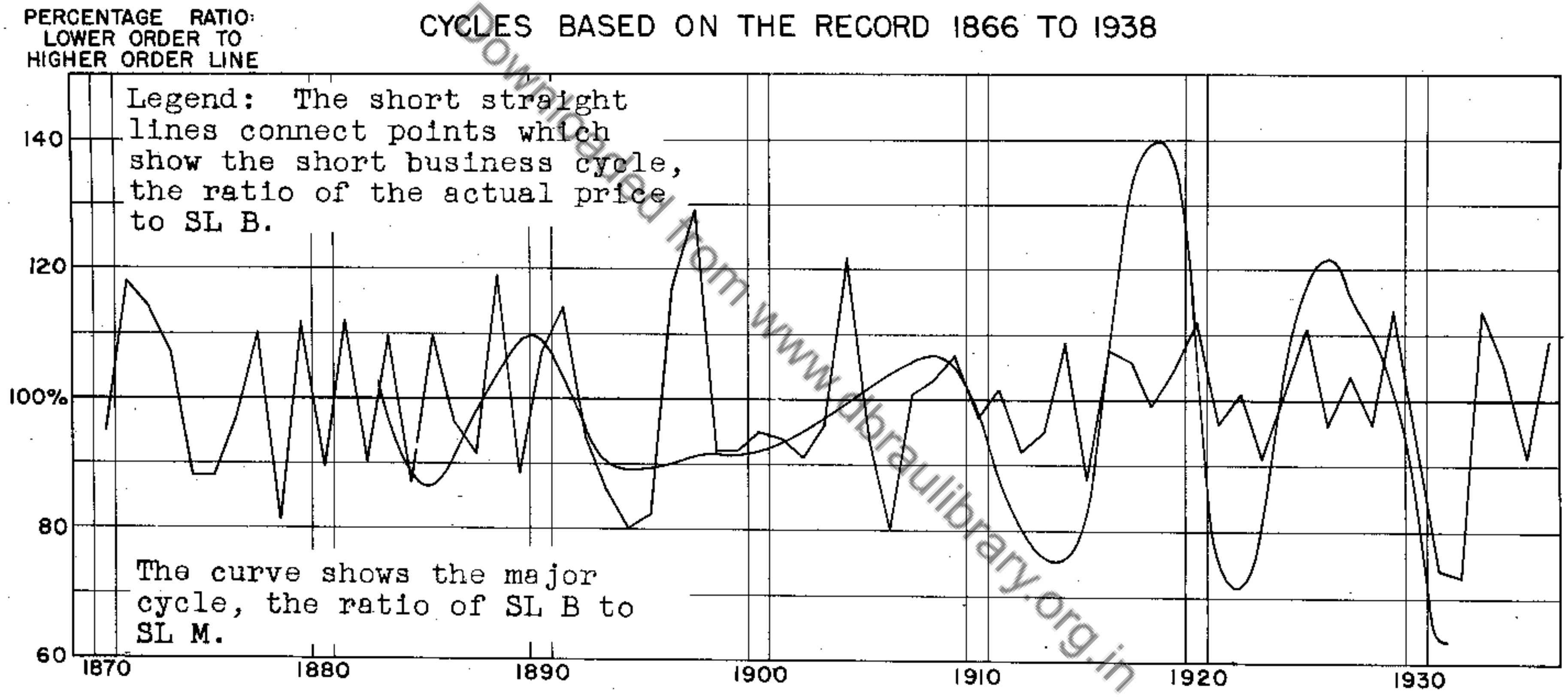


CHART 10.
UNITED STATES DECEMBER FARM PRICES OF WHEAT, 1866 TO 1915.
WITH TRENDS AND CYCLES FROM KUZNETS.

Part a, The annual prices, the intermediate trend (calculated from Kuznets' figures), and the parabolic trend.

Part b, The short business cycle and the major cycle.

Note: a correction has been made for 1881, from the value given by Kuznets in his column V, for the short business cycle.

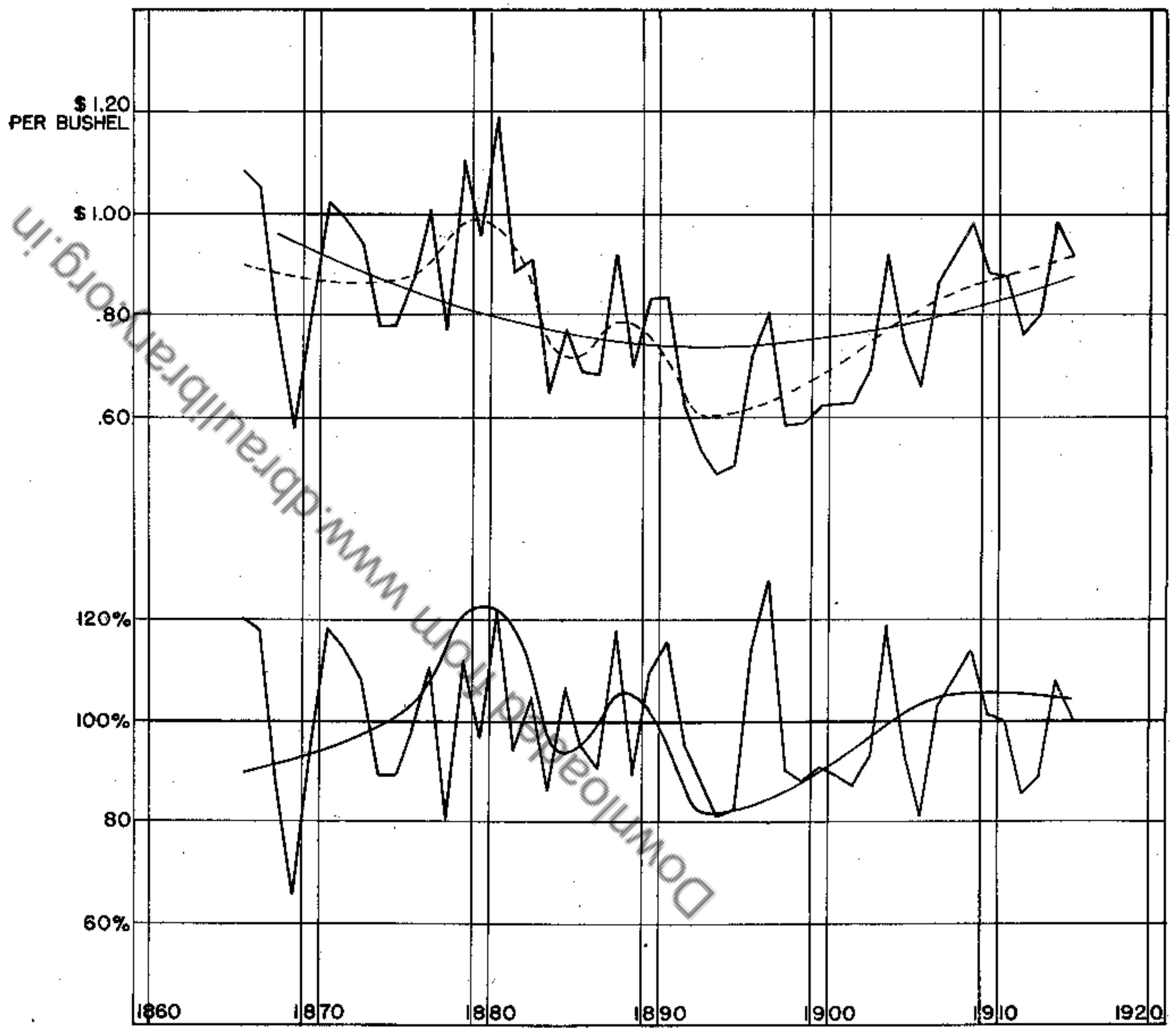


Table G. UNITED STATES DECEMBER FARM PRICES OF WHEAT

Calculation of standard measures of the two orders of cycles.

Based on the record 1866 to 1938. Columns numbered as in Tables D and E. Three pages.

1 Year	2 (or 4) Wheat Price	12 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(cents per bushel)			The short business cycle			The major cycle			
1866	108.4	(87.0)	(106.2)							1866
67	105.1	(86.5)	(105.9)							67
68	77.7	(86.5)	(105.5)							68
69	57.5	(86.0)	(105.1)							69
1870	82.1	(86.5)	(104.8)							1870
71	102.5	87.0	(104.0)	118	+18	324				71
72	99.1	87.0	(103.3)	114	+14	196				72
73	94.0	87.5	(102.7)	107	+ 7	49				73
74	77.6	88.0	(101.9)	88	-12	144				74
75	77.9	89.0	(101.0)	88	-12	144				75
76	86.9	90.0	(100.0)	97	-3	9				76
77	100.9	91.5	(98.9)	110	+10	100				77
78	76.5	94.0	(97.7)	81	-19	361				78
79	110.6	99.0	(96.0)	112	+12	144				79
1880	95.1	106.5	(94.4)	89	-11	121				1880
81	119.2	106.0	(92.5)	112	+12	144				81
82	88.4	98.0	(90.3)	90	-10	100				82
83	91.1	82.5	97.4	110	+10	100	85	-15	225	83
84	64.5	74.5	84.8	87	-13	169	88	-12	144	84
85	77.1	70.0	81.5	110	+10	100	86	-14	196	85
86	68.7	71.5	78.2	96	- 4	16	91	- 9	81	86
87	68.1	75.0	75.7	91	- 9	81	99	- 1	1	87
88	92.6	77.5	73.0	119	+19	361	106	+ 6	36	88
89	69.5	79.0	71.0	88	-12	144	111	+11	121	89
1890	83.3	77.5	69.3	107	+ 7	49	112	+12	144	1890
91	83.4	73.0	68.0	114	+14	196	107	+ 7	49	91
92	62.2	66.5	67.2	94	- 6	36	99	- 9	81	92
93	53.5	62.0	66.5	86	-14	196	93	- 7	49	93
94	48.9	61.0	66.2	80	-20	400	92	- 8	64	94
95	50.3	61.0	66.2	82	-18	324	92	- 8	64	95
96	71.7	61.5	66.6	117	+17	289	92	- 8	64	96
97	80.9	62.5	66.8	129	+29	841	94	- 6	36	97
1898	58.2	63.5	67.2	92	- 8	64	95	- 5	25	1898

Table G (continued) United States December Farm Prices of Wheat

1 Year	2 (or 4) Wheat Price	12 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
		(cents per bushel)								
99	58.6	64.0	67.9	92	- 8	64	94	- 6	36	99
1900	62.0	65.0	69.0	95	- 5	25	94	- 6	36	1900
01	62.6	66.5	70.0	94	- 6	36	95	- 5	25	01
02	63.0	69.0	71.3	91	- 9	81	97	- 3	9	02
03	69.5	72.5	73.1	96	- 4	16	99	- 1	1	03
04	92.4	75.5	75.0	122	+22	484	101	+ 1	1	04
05	74.6	79.0	76.6	94	- 6	36	103	+ 3	9	05
06	66.2	82.5	78.5	80	-20	400	105	+ 5	25	06
07	86.5	86.0	81.5	101	+ 1	1	106	+ 6	36	07
08	92.2	89.5	84.5	103	+ 3	9	106	+ 6	36	08
09	98.4	92.0	87.8	107	+ 7	49	105	+ 5	25	09
1910	88.3	91.0	92.4	97	- 3	9	99	- 1	1 end of the short period for calculation of sd.	1910
11	87.4	85.5	97.5	102	+ 2	4	88	-12	144	11
12	76.0	83.0	104.0	92	- 8	64	80	-20	400	12
13	79.9	84.5	112.0	95	- 5	25	75	-25	625	13
14	98.6	90.5	122.0	109	+ 9	81 end of the short period for calculation of sd.	74	-26	676	14
15	91.9	105.0	132.5	88	-12	144	79	-21	441	15
16	160.3	148.0	143.0	108	+ 8	64	104	+ 4	16	16
17	200.8	189.0	150.0	106	+ 6	36	126	+26	676	17
18	204.2	206.5	156.0	99	- 1	1	132	+32	1024	18
19	216.3	205.5	158.0	105	+ 5	25	130	+30	900	19
1920	182.6	163.0	156.5	112	+12	144	104	+ 4	16	1920
21	103.0	107.0	149.0	96	- 4	16	72	-28	784	21
22	96.6	95.5	135.5	101	+ 1	1	71	-29	841	22
23	92.6	102.0	124.0	91	- 9	81	82	-18	324	23
24	124.7	123.0	114.0	101	+ 1	1	108	+ 8	64	24
25	143.7	130.0	106.6	111	+11	121	122	+22	484	25
26	121.7	126.5	101.0	96	- 4	16	125	+25	625	26
27	119.0	114.0	96.3	104	+ 4	16	118	+18	324	27
28	99.8	104.0	92.3	96	- 4	16	113	+13	169	28
29	103.4	91.0	88.5	114	+14	196	103	+ 3	9	29
1930	67.0	72.0	85.4	93	- 7	49	84	-16	256	1930
31	39.0	53.0	(82.1)	74	-26	676				31
32	37.9	52.0	(79.5)	73	-27	729				32
33	74.1	65.0	(76.6)	114	+14	196				33
34	84.8	81.0	(74.2)	105	+ 5	25				34
35	83.2	91.0	(72.0)	91	- 9	81				35
36	102.6	(94.5)	(69.8)							36
37	96.3	(91.0)	(67.5)							37
1938	56.1	(82.5)	(65.6)							1938

$\Sigma(d^2) = 10418$

$sd = \sqrt{\frac{10418}{48}} - 14.7\%$ in the major cycle. (the full period)

$\Sigma(d^2) = 9220$

$sd = \sqrt{\frac{9220}{65}} = 11.9\%$, in the short business cycle. (the full period)

(Table G is concluded on the next page)

Table G (concluded) Wheat Prices, Standard measures of the two orders of cycles

Time Lengths of the Phases of the Cycles

Of actual about SL B[28] (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1	-	-	1.7	1.4
2	1.1	2.0	1.0	1.4
3	1.1	.5	.5	.6
4	.4	.5	.5	.5
5	.5	.6	.4	.5
6	.5	.5	.5	.6
7	.4	.7	1.0	.6
8	.7	.7	.3	.5
9	1.2	1.1	2.2	1.3
10	1.4	1.0	-	-
11	-	-	3.7	1.6
12	.9	.8	1.0	1.1
13	1.9	.9	.5	.3
14	.4	.4	1.0	1.1
15	.6	.5	.5	.6
16	.7	1.4	.5	.3
17	.7	1.8	.4	.2
18	.6	.4	.6	1.0
19	1.1	.8	.3	.4
20	.4	.6	.5	.4
21	.4	1.0	1.7	.9
22	1.0	.9	.6	.5
23	1.0	1.0	-	-
Average Length	.81 yr.	.86 yr.	.92 yr.	.75 yr.

Total length of typical short business cycle 3.21 yrs.

Of SL B about SL M[33] (the major cycle)

1	2.7 yrs.	2.5	2.2	2.1
2	2.0	2.1	4.5	8.2
3	4.3	1.7	2.8	2.5
4	3.2	2.0	1.3	1.8
5	1.7	3.3	2.7	2.3
6	2.2	-	-	-
Average Length	2.68 yrs.	2.32 yrs.	2.70 yrs.	3.38 yrs.

Total length of typical major cycle 11.08 yrs.

Percentage Deviations at Peaks and Troughs

Of actual from SL B[29] (the short business cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1869	-33%
1871-72	+16%	1874-75	12
77	10	78	19
79	12	80	11
81	12	82	10
83	10	84	13
85	10	87	9
88	19	89	12
91	14	94	20
97	29	1901-02	6
1904	22	06	20
09	7	10	3
11	2	12	8
14	9	15	12
16	8	18	1
19	5	21	4
22	1	23	9
25	11	26	4
27	4	28	4
29	14	1931-32	17
1933-34	9	1935	9
1936-37	8		
Average deviation in the short business cycle	+11.0 % at peak		-11.2 % at trough

Of SL B from SL M[34] (the major cycle)

1880-81	+17 %	1885	-14 %
1889	9	1896	10
1908-09	6	1913	24
1919	37	1922	29
1925-26	20	1931-32	35
1936	36		
Average deviation in the major cycle	+20.8% at peak		-22.4% at trough

Section 2. WHEAT PRODUCTION.

COMPARISON of the trend line secured by Kuznets, with the smoothing line, SL M (millions of bushels)

	Kuznets' Trend	SL M
1866	205.5	(173)
1870	245.0	(233)
1875	302.1	329
1880	366.3	390
1885	435.7	440
1890	507.9	494
1895	580.0	551
1900	649.2	610
1905	713.1	663
1910	769.9	718
1915	818.9	776
1920	860.0	838
1925	893.6	855

Kuznets' equation:

$$y = \frac{1012.8}{1 + 10^{(0.49609 - 0.12464x)}}$$

x in units of five years; origin at 1870.

THE early values of SL M have been placed in parentheses because they fall in the terminal half-cycle. The last two tabulated values of SL M, those for 1920 and 1925, begin to show the effect of the down-pull resulting from the very much reduced production of wheat in the United States in the late 1920's and the 1930's. Kuznets' figures were based on evidence which stopped with 1924, and he, of course, had not witnessed the down-pull of the late 1920's.

ON the whole, Kuznets' trend line and SL M agree remarkably well despite the slight discrepancy that has been pointed out for 1920 to 1925, and the further fact that SL M is somewhat lower than the trend line during the period 1895 to 1915.

The values of the standard deviation, by the several calculations:

	From Kuznets' figures, based on period 1866 to 1924	Figures secured by the method of smoothing by stages	
		based on period 1866 to 1924	based on period 1866 to 1938
The short business cycle	11.0%	11.0%	10.5%
Years included (omit terminal half-cycles)	full	1868 to 1923	1868 to 1936
The major cycle	9.3%	7.7%	7.0%
Years included (omit terminal half-cycles)	full	1873 to 1916	1873 to 1931

Table H. WHEAT PRODUCTION IN THE UNITED STATES, 1866 TO 1938

Two stages of smoothing. Columns numbered as in Tables B and C.

Source: Reports of the U. S. Department of Agriculture.

Part a (six pages)

1 Year	2(or 4)a Wheat Production millions bushels	2b(or 4b) logarithm	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Middle	9 End	10 Length in Years	11a Moving Total of 'Logs
1866	152.0	2.18184							
			r_1						
67	212.4	2.32715	p_1						
			f_1	$r_{1,2}$	1867	1867-68	1868	2	4.67740
68	224.0	2.35025	t_1	$p_{1,2}$	1867	1868	1869	3	7.09254
			r_2	$f_{1,2}$	1868	1868-69	1869	2	4.76539
69	260.1	2.41514	p_2						
				$t_{1,2}$	1868	1869-70	1871	4	9.50117
1870	235.9	2.37273	f_2						
				$r_{2,3}$	1869	1870-71	1872	4	9.54886
71	230.7	2.36305	t_2						
				$p_{2,3}$	1869	1871-72	1874	6	14.48672
72	250.0	2.39794		$f_{2,3}$	1870	1872	1874	5	12.07158
			r_3						
73	281.3	2.44917							
				$t_{2,3}$	1871	1873-74	1876	6	14.62588
74	308.1	2.48869	p_3						
			f_3						
75	292.1	2.46553		$r_{3,4}$	1873	1875	1877	5	12.42623
76	289.4	2.46150	t_3						
77	364.2	2.56134	r_4	$p_{3,4}$	1874	1877	1879	6	15.29624
				$f_{3,4}$	1875	1877-78	1880	6	15.50530
78	420.1	2.62335							
79	496.4	2.69583							
			p_4	$t_{3,4}$	1878	1879-80	1881	6	15.62331
1880	498.6	2.67775	f_4	$r_{4,5}$	1878	1880	1881	4	10.58047
81	383.3	2.58354	t_4	$p_{4,5}$	1880	1881	1882	3	7.98389
			r_5	$f_{4,5}$	1881	1881-82	1882	2	5.28514
82	504.2	2.70260	p_5	$t_{4,5}$	1881	1882	1883	3	7.91053
			f_5	$r_{5,6}$	1882	1882-83	1883	2	5.32699
83	421.1	2.62439	t_5	$p_{5,6}$	1882	1883	1884	3	8.03694
			r_6	$f_{5,6}$	1883	1883-84	1884	2	5.33434
84	512.8	2.70995	p_6	$t_{5,6}$	1883	1884	1885	3	7.88713
			f_6	$r_{6,7}$	1884	1884-85	1885	2	5.26274
85	357.1	2.55279	t_6	$p_{6,7}$	1884	1885	1886	3	7.92285
86	457.2	2.66011	r_7	$f_{6,7}$	1885	1886	1887	3	7.87215
			p_7	$t_{6,7}$	1885	1886-87	1888	4	10.49114
87	456.3	2.65925		$r_{7,8}$	1886	1887	1888	3	7.93835
			f_7						
1888	415.9	2.61899	t_7	$p_{7,8}$	1887	1888	1889	3	7.91613

Table H, Part a (continued) Wheat Production, two stages of smoothing

11b	11c	12a	12b	13	20a	20b	24	1
Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Wheat Production (millions bushels)	Smoothing Line B millions bushels	Logarithm	Phase Point (major cycle)	Second Approx. to SL M millions bushels	Logarithm	Final SL M (millions bushels)	Year
		(175)	2.24304				173	1866
		(205)	2.31175	r_1	183	2.26245	(186)	67
2.33870	218.1							
2.36418	231.3	230	2.36173	p_1	197	2.29447	(201)	68
2.38270	241.4							
		240	2.38021		211	2.32428	(217)	69
2.37529	237.3							
		243	2.38561	f_1	229	2.35984	(233)	1870
2.38722	243.9							
		248	2.39445		246	2.39093	(251)	71
2.41445	259.7							
2.41431	259.6	255	2.40654		265	2.42325	(271)	72
		265	2.42325	t_1	283	2.45179	292	73
2.43764	273.9							
		280	2.44716		300	2.47712	312	74
2.48524	305.7	300	2.47712		316	2.49969	329	75
		330	2.51851	r_2	333	2.52244	345	76
2.54937	354.3	367	2.56467		347	2.54033	357	77
2.58421	383.9							
		400	2.60206		358	2.55388	370	78
		425	2.62839		369	2.56703	380	79
2.60388	401.7							
2.64512	441.7	440	2.64345	p_2	380	2.57978	390	1880
2.66127	458.4	450	2.65321		390	2.59106	400	81
2.64257	439.1							
2.63684	433.4	450	2.65321		400	2.60206	410	82
2.66329	460.6							
2.67898	477.5	445	2.64836		412	2.61490	419	83
2.66717	464.7							
2.62904	425.6	440	2.64345	f_2	424	2.62737	429	84
2.63137	427.9							
2.64095	437.5	433	2.63649		435	2.63849	440	85
2.62405	420.8	425	2.62839		447	2.65031	450	86
2.62278	419.5							
2.64611	442.7	425	2.62839		459	2.66181	462	87
2.62871	425.3	429	2.63246		471	2.67302	472	1888

(Table H, Part a, is continued on next page.)

Table H, Part a, (continued) Wheat Production, two stages of smoothing

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a
Year	Wheat Production		Phase Point	Cycle	Yearly Figures Included in the Cycle			Length in Years	Moving Total of Logs
	millions bushels	Logarithm			Begin	Middle	End		
89	434.4	2.63789	r_8 p_8	$f_{7,8}$ $t_{7,8}$	1888 1888	1888-89 1889	1889 1890	2 3	5.25688 7.83449
1890	378.1	2.57761	f_8 t_8	$r_{8,9}$ $p_{8,9}$	1889 1889	1889 1890	1890 1891	2 3	5.21550 7.98228
91	584.5	2.76678	r_9 p_9	$f_{8,9}$	1890	1891	1892	3	8.06702
92	528.0	2.72263		$t_{8,9}$ $r_{9,10}$	1890 1891	1891-92 1892	1893 1894	4 4	10.69806 10.83352
93	427.6	2.63104	f_9 t_9	$p_{9,10}$	1891	1893	1895	5	13.58901
94	516.5	2.71307	r_{10}	$f_{9,10}$	1893	1894	1895	3	8.09960
95	569.5	2.75549	p_{10}	$t_{9,10}$	1893	1894-95	1896	4	10.83536
96	544.2	2.73576	f_{10} t_{10}	$r_{10,11}$	1894	1895-96	1897	4	10.98986
97	610.3	2.78554	r_{11}	$p_{10,11}$	1895	1896-97	1898	4	11.16452
98	772.2	2.88773	p_{11}	$f_{10,11}$ $t_{10,11}$	1896 1896	1897-98 1898	1898 1900	3 5	8.40903 13.99266
99	636.1	2.80353	f_{11}	$r_{11,12}$	1898	1899	1900	3	8.47136
1900	602.7	2.78010	t_{11}	$p_{11,12}$	1898	1899-00	1901	4	11.36822
01	788.6	2.89686	r_{12} p_{12}	$f_{11,12}$	1899	1900-01	1902	4	11.34071
02	724.8	2.86022		$t_{11,12}$	1900	1902	1904	5	14.13518
03	663.9	2.82210	f_{12}	$r_{12,13}$	1901	1902-03	1904	4	11.35508
04	596.9	2.77590	t_{12}	$p_{12,13}$	1901	1903-04	1906	6	17.09547
05	726.8	2.86141	r_{13}	$f_{12,13}$	1903	1904-05	1906	4	11.33839
06	756.8	2.87898	p_{13}	$t_{12,13}$	1904	1905-06	1907	4	11.32111
07	638.0	2.80482	f_{13}	$r_{13,14}$	1905	1906-07	1908	4	11.35457
08	644.7	2.80936	t_{13}	$p_{13,14}$ $f_{13,14}$	1906 1907	1907-08 1908	1909 1909	4 3	11.33851 8.45953
09	700.4	2.84535	r_{14} p_{14}	$t_{13,14}$	1908	1909	1910	3	8.45755
1910	635.1	2.80284	f_{14}	$r_{14,15}$	1908	1910	1911	4	11.25085
11	621.3	2.79330	t_{14}	$p_{14,15}$ $f_{14,15}$	1909 1910	1910-11 1911	1911 1912	3 3	8.44149 8.45964
12	730.3	2.86350	r_{15} p_{15}	$t_{14,15}$	1911	1912	1912	2	5.65680
13	763.4	2.88275	f_{15} t_{15}	$r_{15,16}$	1912	1912-13	1913	2	5.74625

Table H, Part a, (continued) Wheat Production, two stages of smoothing

11b mca of Logs	11c mc Geom Mean of Wheat Production (millions bu.)	12a SL B millions bushels	12b Log of SL B	13 Phase Point	20a Second Approximation to SL M	20b	24 Final SL M millions bushels	1 Year
					millions bushels	logarithm		
2.62844 2.61147	425.0 408.8	435	2.63849	t_2	483	2.68395	483	1889
2.60775 2.66076	405.3 457.9	445	2.64836		495	2.69461	494	1890
2.68900	488.7	460	2.66276		507	2.70501	505	91
2.67451 2.70838	472.6 511.0	478	2.67943		517	2.71349	516	92
2.71780	522.2	500	2.69897		528	2.72263	528	93
2.69986	501.0	525	2.72016		538	2.73078	539	94
2.70884	511.5	555	2.74429	r_3	548	2.73878	551	95
2.74746	559.1	590	2.77085		560		563	96
2.79113	618.2	620	2.79239		572		576	97
2.80301 2.79853	635.3 628.8	645	2.80956		587		588	98
2.82378	666.5	665	2.82282		600		600	99
2.84205	659.1	675	2.82930	p_3	613		610	1900
2.83517	684.2	683	2.83442		630		621	01
2.82703	671.5	690	2.83885		645		631	02
2.83877	689.9	695	2.84198		660		642	03
2.84924	706.7	692	2.84011		673		653	04
2.83459	688.3	688	2.83759		681		663	05
2.83027	676.5	680	2.83251	f_3	690		673	06
2.83864	689.7	672	2.82737		698		684	07
2.83462 2.81984	683.3 660.4	667	2.82413		705		696	08
2.81918	659.5	660	2.81954		713		707	09
2.81271	649.7	654	2.81558	t_3	722		718	1910
2.81383 2.81988	651.4 660.5	664	2.82217		731	2.86392	730	11
2.82840	673.6	710	2.85126	r_4	742	2.87041	741	12
2.87312	746.7	780	2.89209		752	2.87622	752	13

(Table H, Part a, is continued on next page)

Table H, Part a (concluded) Wheat Production, two stages of smoothing.

1	2(or 4) a	2b	5	6	7	8	9	10	11a
Year	Wheat Production millions bushels	Logarithm	Phase Point	Cycle	Yearly Figures Included in the Cycle Begin	Middle	End	Length in Years	Moving total of Logs
			r_{16}	$p_{15,16}$	1912	1913-14	1915	4	11.70719
1914	891.0	2.94988		$f_{15,16}$	1913	1914	1915	3	8.84369
15	1025.8	3.01106	p_{16}	$t_{15,16}$	1913	1915	1916	4	11.64735
			f_{16}	$r_{16,17}$	1914	1915-16	1917	4	11.56853
16	636.3	2.80366							
			t_{16}						
17	636.7	2.80393		$p_{16,17}$	1915	1917	1919	5	14.57898
18	921.4	2.96445	r_{17}	$f_{16,17}$	1916	1918	1919	4	11.56792
19	968.0	2.99588	p_{17}	$t_{16,17}$	1917	1919	1921	5	14.59601
				$r_{17,18}$	1918	1919-20	1921	4	11.79208
1920	833.0	2.92065	f_{17}						
				$p_{17,18}$	1919	1920-21	1922	4	11.76595
21	814.9	2.91110	t_{17}	$f_{17,18}$	1920	1921	1922	3	8.77007
			r_{18}						
22	867.6	2.93832	p_{18}	$t_{17,18}$	1921	1922	1923	3	8.75110
			f_{18}	$r_{18,19}$	1922	1922-23	1923	2	5.84000
23	797.4	2.90168	t_{18}	$p_{18,19}$	1922	1923	1924	3	8.77581
			r_{19}	$f_{18,19}$	1923	1923-24	1924	2	5.83749
24	862.6	2.93581	p_{19}	$t_{18,19}$	1923	1924	1925	3	8.66795
			f_{19}	$r_{19,20}$	1924	1924-25	1925	2	5.76627
25	676.8	2.83046	t_{19}						
				$p_{19,20}$	1924	1926	1928	5	14.58951
26	833.5	2.92091	r_{20}						
				$f_{19,20}$	1925	1926-27	1928	4	11.65370
27	874.7	2.94186		$t_{19,20}$	1925	1927	1929	5	14.56868
				$r_{20,21}$	1926	1927-28	1929	4	11.83722
28	913.0	2.96047	p_{20}						
			f_{20}						
29	822.2	2.91498	t_{20}						
			r_{21}	$p_{20,21}$	1928	1929-30	1930	3	8.82469
1930	889.7	2.94924							
				$f_{20,21}$	1929	1930-31	1932	4	11.70635
31	932.2	2.96951	p_{21}	$t_{20,21}$	1929	1931	1933	5	14.42981
32	745.8	2.87262		$r_{21,22}$	1930	1932	1934	5	14.23615
			f_{21}						
33	529.0	2.72346		$p_{21,22}$	1931	1933	1935	5	14.08369
			t_{21}						
34	526.4	2.72132		$f_{21,22}$	1933	1934	1935	3	8.24156
			r_{22}						
35	626.3	2.79678	p_{22}	$t_{21,22}$	1934	1935	1936	3	8.31523
			f_{22}	$r_{22,23}$	1935	1935-36	1936	2	5.59391
36	626.8	2.79713	t_{22}						
			r_{23}	$p_{22,23}$	1935	1936-37	1938	4	11.50554
37	875.7	2.94235	p_{23}						
1938	931.7	2.96928							

Table H, Part a (concluded) Wheat Production, two stages of smoothing.

11b mca of Logs	11c mc Geom. Mean of Wheat Production (millions bu.)	12a SL B millions bushels	12b Log of SL B	13 Phase Point	20a Second Approximation to SL M	20b	24 Final SL M millions bushels	1 Year
					millions bushels	logarithm		
2.92679	844.9							
2.94123	873.4	805	2.90580		765	2.88366	763	1914
2.91183	816.3	817	2.91381		776	2.88986	776	15
2.89213	780.1							
		826	2.91593	p_4	788	2.89653	788	16
2.91579	823.7	835	2.92169		798	2.90200	800	17
2.89198	779.8	842	2.92531		808	2.90741	812	18
2.91920	830.2	848	2.92840		819	2.91328	827	19
2.94802	887.2							
		850	2.92952		829	2.91856	838	1920
2.94148	873.9							
2.92335	838.2	845	2.92686		832	2.92012	845	21
2.91703	826.1	838	2.92324	f_4	836	2.92221	850	22
2.92000	831.8							
2.92527	841.9	820	2.91381		840	2.92428	853	23
2.91874	829.4							
2.88931	775.0	810	2.90849		842	2.92531	855	24
2.88314	764.1							
		805	2.90580	t_4	844	2.92634	855	25
2.91790	827.7	820	2.91381		842	2.92531	851	26
2.91343	819.3							
2.91374	819.9	845	2.92686	r_5	836	2.92221	848	27
2.93455	860.1							
		870	2.93952		820	2.91381	832	28
		880	2.94448	p_5	803	2.90472	817	29
2.94156	874.1	870	2.93952		783	2.89376	797	1930
2.93545	861.9	815	2.91116	f_5	760	2.88081	777	31
2.88596	796.1							
2.84723	703.4	710	2.85126		735	2.86629	(758)	32
2.81673	655.7	575	2.75967		707	2.84942	(737)	33
2.74718	558.7	560	2.74819	t_5	677	2.83059	(718)	34
2.77174	591.2	590	2.77085		645	2.80956	(697)	35
2.79695	626.5							
		670	2.82608		618	2.79099	(678)	36
2.87638	752.3							
		(785)	2.89487	r_6	590	2.77085	(659)	37
		(900)	2.95424				(640)	1938

Table H. WHEAT PRODUCTION

Part b part of the calculations for the second stage of smoothing, including correction for curvature.

Columns numbered as in Tables B and C.

14 Cycle	15 Yearly Figures Included in the Cycle Begin	16 Middle	17 End	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (first application -millions bushels)	20a (to Table Ha) Second Approximation to SL M	20b Log of Second Approximation	21a Moving Total of Logs of Second Approximation	21b mca of Logs of Second Approx.	22 Difference in Logs (column 21 b Minus column 19 b)	23a Adjusted mca of Logs	23b Adjusted Moving Cyclical Geometric Mean (millions bushels)	24 (to Table Ha) Final SL M	16 Year
$r_{1,2}$	1867	1872	1876	10	24.10633	2.41063	257.4			24.00626	2.40063	-.01000	2.42063	263.4		1872
$p_{1,2}$	1868	1874	1880	13	32.23315	2.47947	301.6			31.98483	2.46037	-.01910	2.49857	315.2		1874
$f_{1,2}$	1870	1877	1884	15	38.08944	2.53930	346.2			37.80147	2.52010	-.01920	2.55850	361.8		1877
$t_{1,2}$	1873	1881	1889	17	44.06706	2.59218	391.0			43.93503	2.58441	-.00777	2.59995	398.1		1881
$r_{2,3}$	1877	1886	1895	19	50.35499	2.65026	447.0			50.28929	2.64680	-.00346	2.65372	447.0		1886
$p_{2,3}$	1881	1890	1900	20	53.94134	2.69707	497.4							497.4		1890
$f_{2,3}$	1884	1895	1906	23	63.01202	2.73965	549.1				Curvature so slight that these mca's were not reiterated.			549.1		1895
$t_{2,3}$	1890	1900	1910	21	58.49097	2.78528	609.9							609.9		1900
$r_{3,4}$	1896	1904	1912	17	48.01043	2.82414	667.0							667.0		1904
$p_{3,4}$	1900	1908	1916	17	48.44244	2.84956	707.2							707.2		1908
$f_{3,4}$	1907	1914	1922	16	46.14270	2.88392	765.5							765.5		1914
$t_{3,4}$	1911	1918	1925	15	43.58418	2.90561	804.7			43.54011	2.90267	-.00294	2.90855	804.7		1918
$r_{4,5}$	1913	1919-20	1927	15	43.75142	2.91676	825.5			43.65330	2.91022	-.00654	2.92330	838.1		1919-20
$p_{4,5}$	1916	1922-23	1929	14	40.92372	2.92312	837.8			40.82209	2.91586	-.00726	2.93038	851.9		1922-23
$f_{4,5}$	1923	1926-27	1930	8	23.39229	2.92404	839.5			23.33574	2.91697	-.00707	2.93111	853.3		1926-27
$t_{4,5}$	1925	1929-30	1933	9	26.09208	2.89912	792.7			26.08267	2.89807	-.00105	2.90017	792.7		1929-30
$r_{5,6}$	1927	1932	1937	11	31.51246	2.86477	732.4			31.43301	2.85755	-.00722	2.87199	732.4		1932

Note: On the work sheet these calculations were extended vertically and incorporated with part a. See note on Table F, Part b.

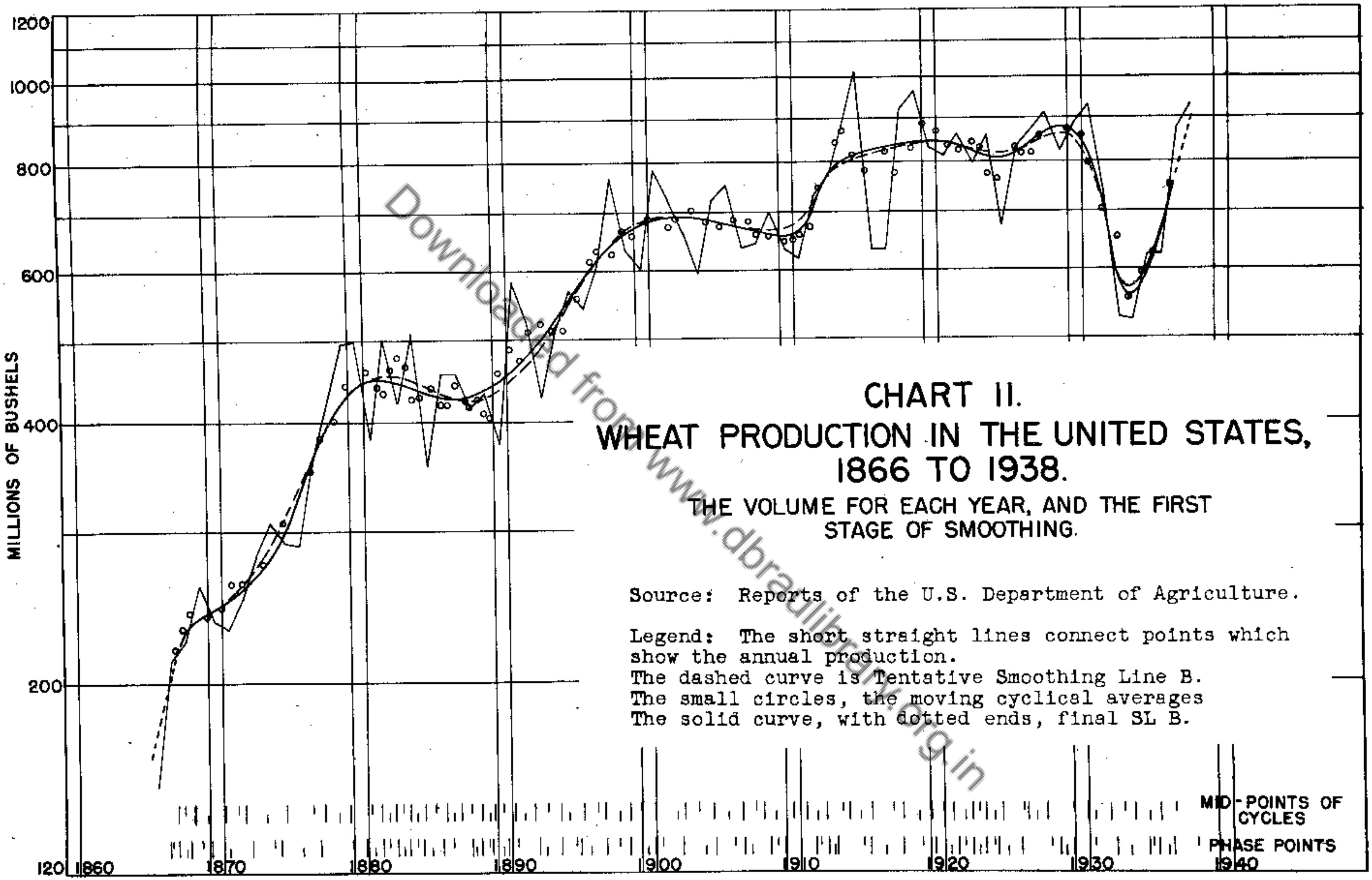


CHART II.
WHEAT PRODUCTION IN THE UNITED STATES, 1866 TO 1938.

THE VOLUME FOR EACH YEAR, AND THE FIRST STAGE OF SMOOTHING.

Source: Reports of the U.S. Department of Agriculture.

Legend: The short straight lines connect points which show the annual production.
The dashed curve is Tentative Smoothing Line B.
The small circles, the moving cyclical averages
The solid curve, with dotted ends, final SL B.

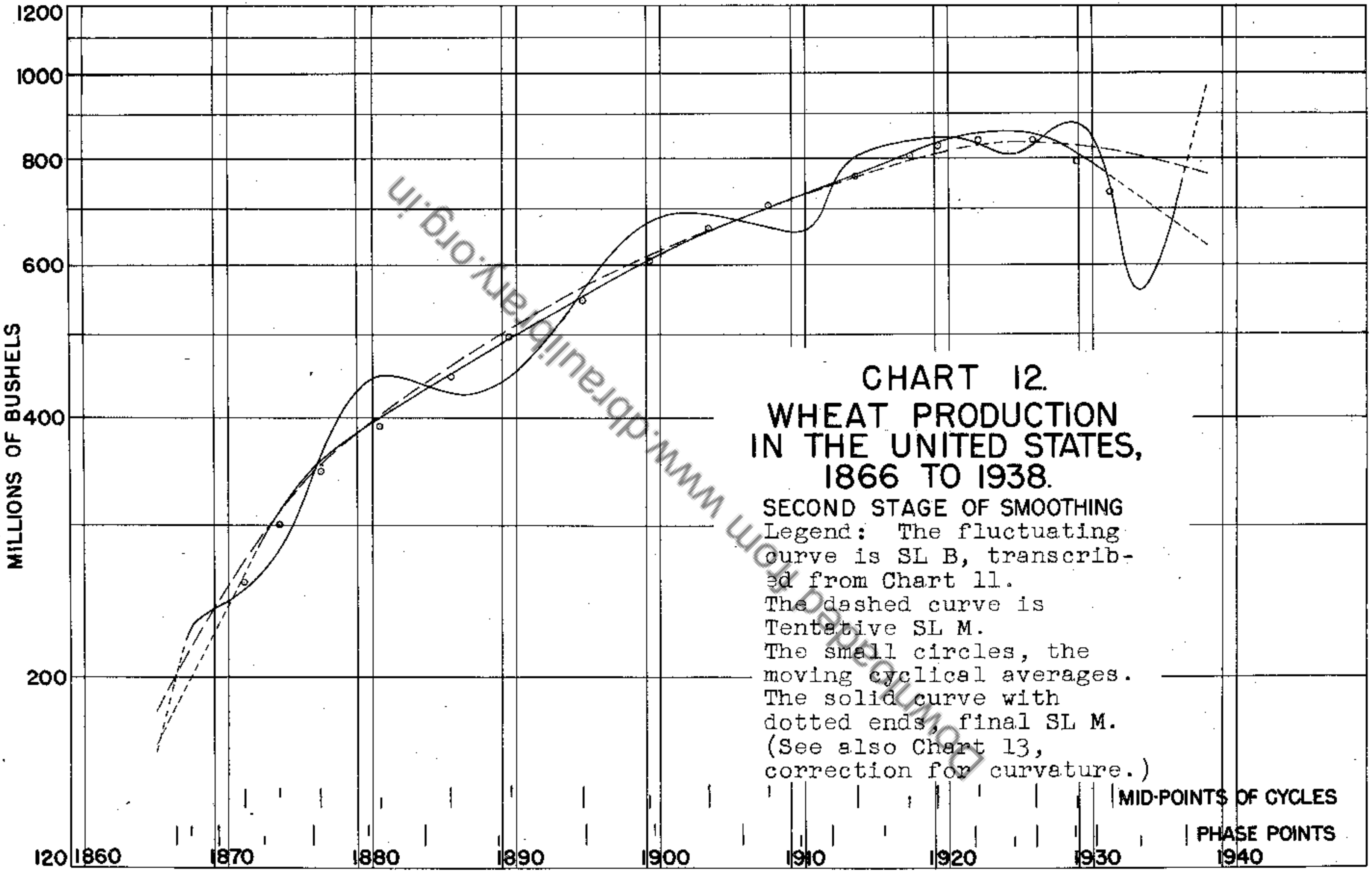


CHART 12.

WHEAT PRODUCTION IN THE UNITED STATES, 1866 TO 1938.

SECOND STAGE OF SMOOTHING

Legend: The fluctuating curve is SL B, transcribed from Chart 11.
The dashed curve is Tentative SL M.
The small circles, the moving cyclical averages.
The solid curve with dotted ends, final SL M.
(See also Chart 13, correction for curvature.)

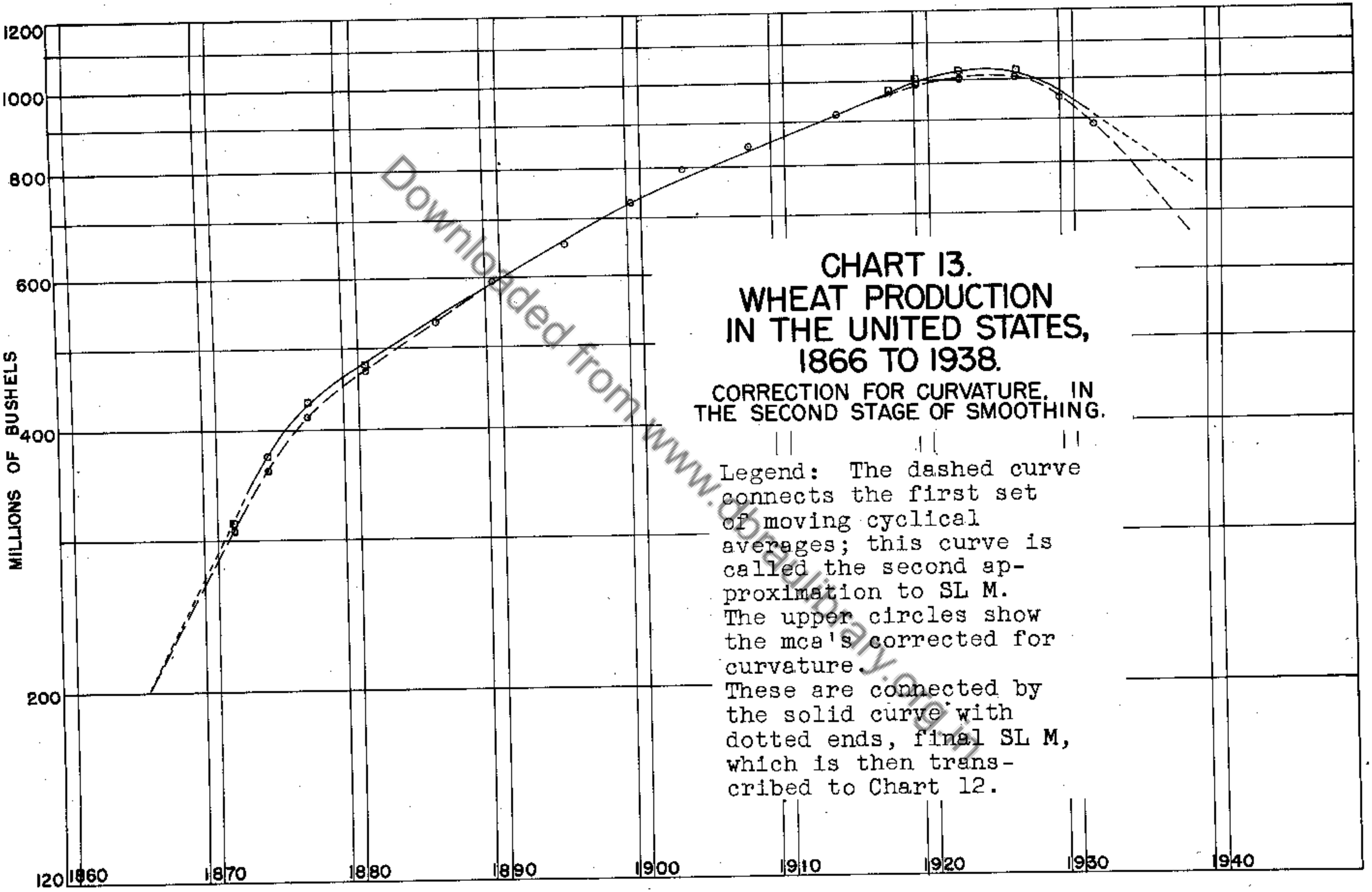


CHART 13.
WHEAT PRODUCTION
IN THE UNITED STATES,
1866 TO 1938.

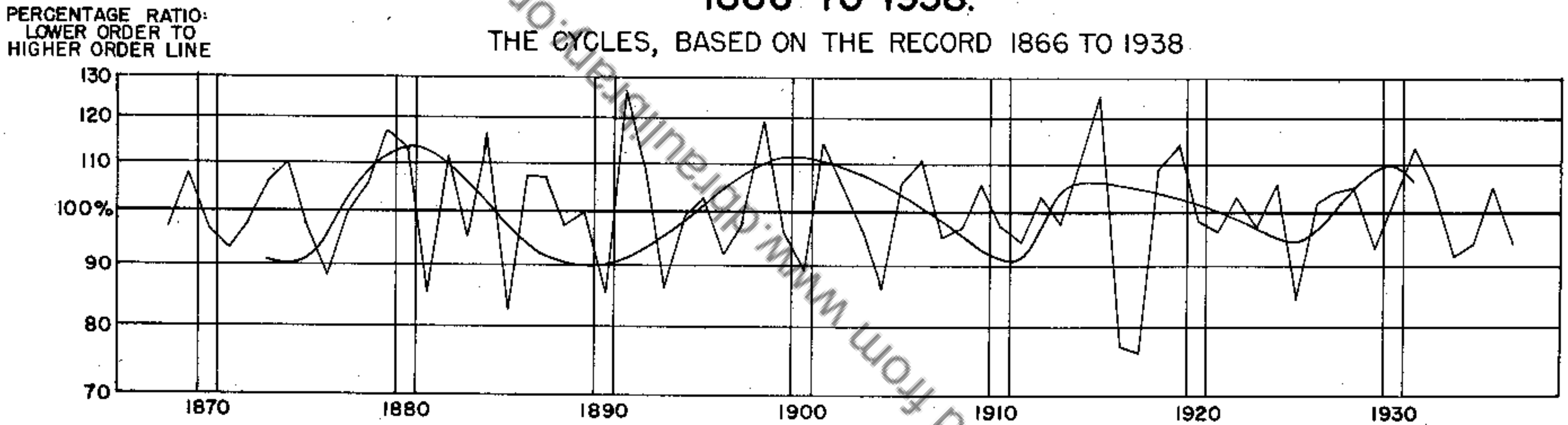
CORRECTION FOR CURVATURE. IN THE SECOND STAGE OF SMOOTHING.

Legend: The dashed curve connects the first set of moving cyclical averages; this curve is called the second approximation to SL M. The upper circles show the mca's corrected for curvature. These are connected by the solid curve with dotted ends, final SL M, which is then transcribed to Chart 12.

CHART 14.

WHEAT PRODUCTION IN THE UNITED STATES 1866 TO 1938.

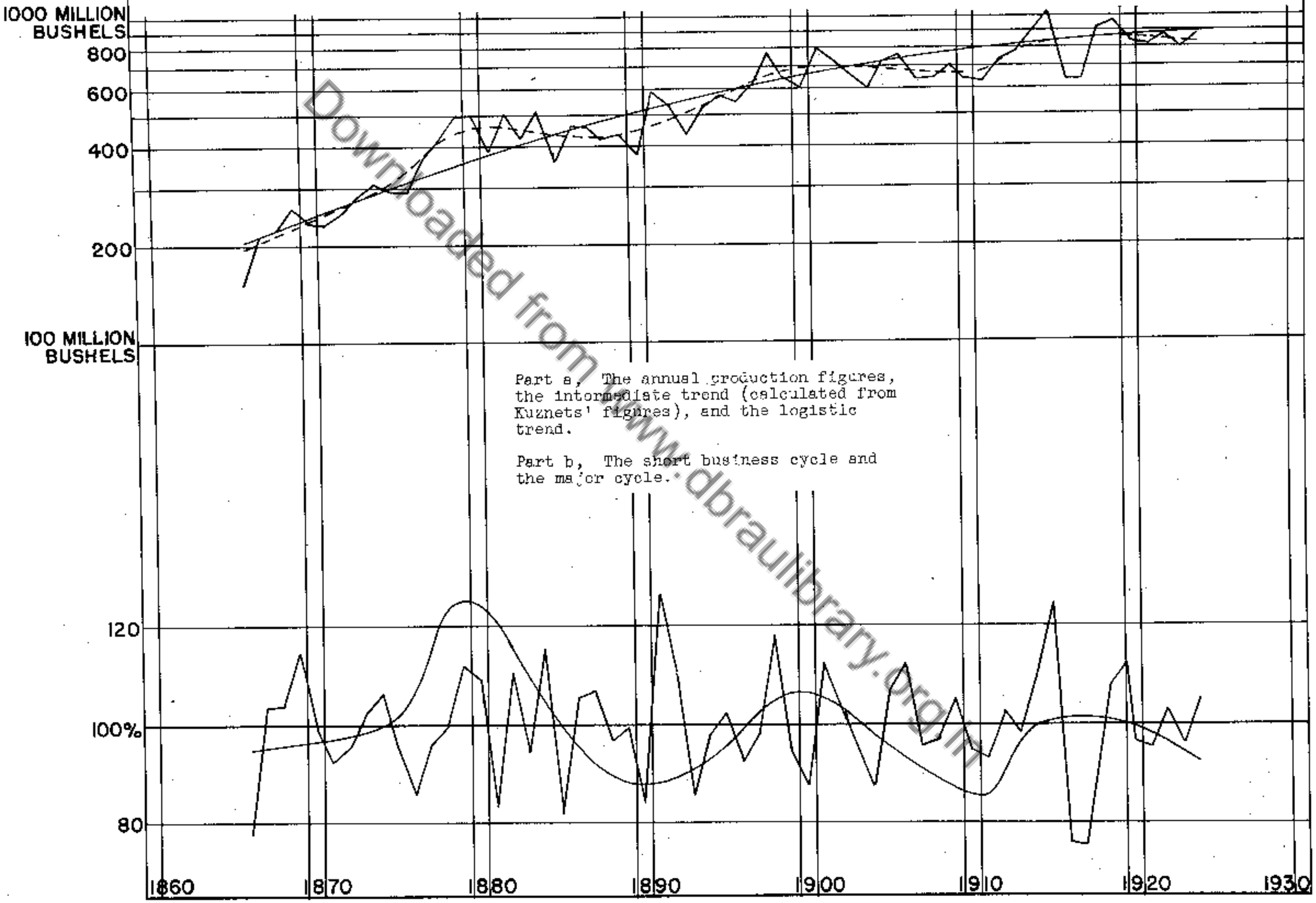
THE CYCLES, BASED ON THE RECORD 1866 TO 1938



Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual production to SL B. The curve shows the major cycle, the ratio of SL B to SL M.

CHART 15. WHEAT PRODUCTION IN THE UNITED STATES, 1866 TO 1924

WITH TRENDS AND CYCLES FROM KUZNETS



Part a, The annual production figures, the intermediate trend (calculated from Kuznets' figures), and the logistic trend.

Part b, The short business cycle and the major cycle.

Table J. WHEAT PRODUCTION IN THE UNITED STATES

Calculation of standard measures of the two orders of cycles.

Based on the record 1866 to 1938. Columns numbered as in Tables D and E. Three pages.

1 Year	2 (or 4)a Wheat Production	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(millions of bushels)				The short business cycle			The major cycle		
1866	152.0									1866
67	212.4									67
68	224.0	230		97	- 3%	9				68
69	260.1	240		108	+ 8	64				69
1870	235.9	243		97	- 3	9				1870
71	230.7	248		93	- 7	49				71
72	250.0	255		98	- 2	4				72
73	281.3	265	292	106	+ 6	36	91	- 9	81	73
74	308.1	280	312	110	+10	100	90	-10	100	74
75	292.1	300	329	97	- 3	9	91	- 9	81	75
76	289.4	330	345	88	-12	144	96	- 4	16	76
77	364.2	367	357	99	- 1	1	103	+ 3	9	77
78	420.1	400	370	105	+ 5	25	108	+ 8	64	78
79	496.4	425	380	117	+17	289	112	+12	144	79
1880	498.6	440	390	113	+13	169	113	+13	169	1880
81	383.3	450	400	85	-15	225	112	+12	144	81
82	504.2	450	410	112	+12	144	110	+10	100	82
83	421.1	445	419	95	- 5	25	106	+ 6	36	83
84	512.8	440	429	117	+17	289	102	+ 2	4	84
85	357.1	433	440	82	-18	324	98	- 2	4	85
86	457.2	425	450	108	+ 8	64	94	- 6	36	86
87	456.3	425	462	107	+ 7	49	92	- 8	64	87
88	415.9	429	472	97	- 3	9	91	- 9	81	88
89	434.4	435	483	100	0	0	90	-10	100	89
1890	378.1	445	494	85	-15	225	90	-10	100	1890
91	584.5	460	505	127	+27	729	91	- 9	81	91
92	528.0	478	516	110	+10	100	93	- 7	49	92
93	427.6	500	528	86	-14	196	95	- 5	25	93
94	516.5	525	539	99	- 1	1	97	- 3	9	94
95	569.5	555	551	103	+ 3	9	100	0	0	95
96	544.2	590	563	92	- 8	64	105	+ 5	25	96
97	610.3	620	576	98	- 2	4	108	+ 8	64	97
1898	772.2	645	588	120	+20	400	110	+10	100	1898

Table J (continued) Wheat Production in the United States

1 Year	2(or 4)a Wheat Production	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(millions of bushels)			The short business cycle			The major cycle			
99	636.1	665	600	96	- 4	16	111	+11	121	99
1900	602.7	675	610	89	-11	121	111	+11	121	1900
01	788.6	683	621	115	+15	225	110	+10	100	01
02	724.8	690	631	105	+ 5	25	109	+ 9	81	02
03	663.9	695	642	96	- 4	16	108	+ 8	64	03
04	596.9	692	653	86	-14	196	106	+ 6	36	04
05	726.8	688	663	106	+ 6	36	104	+ 4	16	05
06	756.8	680	673	111	+11	121	101	+ 1	1	06
07	638.0	672	684	95	- 5	25	98	- 2	4	07
08	644.7	667	696	97	- 3	9	96	- 4	16	08
09	700.4	660	707	106	+ 6	36	93	- 7	49	09
1910	635.1	654	718	97	- 3	9	91	- 9	81	1910
11	621.3	664	730	94	- 6	36	91	- 9	81	11
12	730.3	710	741	103	+ 3	9	96	- 4	16	12
13	733.4	780	752	98	- 2	4	104	+ 4	16	13
14	891.0	805	763	111	+11	121	107	+ 7	49	14
15	1025.8	817	776	125	+25	625	105	+ 5	25	15
16	636.3	826	788	77	-23	529	105	+ 5	25	16
17	636.7	835	800	76	-24	576	104	+ 4	16	17
18	921.4	842	812	109	+ 9	81	104	+ 4	16	18
19	968.0	848	827	114	+14	196	102	+ 2	4	19
1920	833.0	850	838	98	- 2	4	101	+ 1	1	1920
21	814.9	845	845	96	- 4	16	100	0	0	21
22	867.6	838	850	103	+ 3	9	99	- 1	1	22
23	797.4	820	853	97	- 3	9	96	- 4	16	23
24	862.6	810	855	106	+ 6	36	95	- 5	25	24
25	676.8	805	855	84	-16	256	94	- 6	36	25
26	833.5	820	851	102	+ 2	4	96	- 4	16	26
27	874.7	845	848	104	+ 4	16	100	0	0	27
28	913.0	870	832	105	+ 5	25	104	+ 4	16	28
29	822.2	880	817	93	- 7	49	108	+ 8	64	29
1930	889.7	870	797	102	+ 2	4	109	+ 9	81	1930
31	932.2	815	777	114	+14	196	105	+ 5	25	31
32	745.8	710		105	+ 5	25				32
33	529.0	575		92	- 8	64				33
34	526.4	560		94	- 6	36				34
35	626.3	590		106	+ 6	36				35
36	626.8	670		94	- 6	36				36
37	875.7									37
1938	931.7									1938

end of the short period for calculation of sd.

end of the short period for calculation of sd.

$\Sigma(d^2) = 7598$

$sd = \sqrt{\frac{7598}{69}} = 10.5\%$, in the short business cycle.
(the full period)

$\Sigma(d^2) = 2905$

$sd = \sqrt{\frac{2905}{59}} = 7.0\%$, in the major cycle.
(the full period)

(Table J is concluded on the next page.)

Table J (concluded) Wheat Production in the United States

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles

[28] of actual about SL B (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1	-	-	.7	.2
2	.7	.6	1.3	1.6
3	1.5	.5	1.5	1.2
4	2.3	.8	.6	.5
5	.5	.6	.4	.4
6	.6	.4	.6	.7
7	.8	1.2	.3	.4
8	.6	.2	.7	.4
9	.7	1.4	.6	1.0
10	1.0	.3	.7	1.1
11	.9	.8	1.2	.5
12	.5	1.5	1.5	.7
13	1.3	.7	.6	1.3
14	.5	.5	1.0	1.2
15	.3	.5	.5	.2
16	1.8	.5	1.0	1.2
17	1.3	.8	1.2	.5
18	.5	.5	.5	.4
19	.6	.3	.8	.9
20	2.1	.6	.5	.5
21	1.5	1.5	1.0	1.2
22	.3	.4	.6	--
Average Length	1.0 yr.	.7 yr.	.8 yr.	.8 yr.

Total length of typical short business cycle 3.2 yrs.

[33] Of SL B about SL M (the major cycle)

Cycle Number	rp	pf	ft	tr
1	-	-	3.2	3.4
2	3.8	3.9	5.1	6.1
3	4.7	6.1	4.0	2.2
4	3.7	6.3	2.7	1.5
5	2.8	1.4	-	-
Average Length	3.7 yrs.	4.4 yrs.	3.7 yrs.	3.3 yrs.

Total length of typical major cycle 15.3 yrs.

Percentage Deviations at Peaks and Troughs

[29] of actual from SL B (the short business cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1868	- 3%
1869	+ 8%	71	- 7
74	+10	76	12
79	17	81	15
82	12	83	5
84	17	85	18
86	8	88	3
89	0	90	15
91	27	93	14
95	3	96	8
98	20	1900	11
1901	15	04	14
06	11	07	5
09	6	11	6
12	3	13	2
15	25	17	24
19	14	21	4
22	3	23	3
24	6	25	16
28	5	29	7
31	14	33	8
1935	6	1936	6
Average Deviation in the short business cycle	11.0 % at peak		9.4 % at trough

[34] of SL B from SL M (the major cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1873	- 6%
1880	+14%	1889	10
1900	10	1910	10
1915	5	1925	3
1929	7		
Average Deviation in the major cycle	9.0 % at peak		7.2% at trough

Section 3. COTTON PRODUCTION.

COMPARISON of the trend line secured by Kuznets with the smoothing line SL M (thousands of bales)

	Kuznets' Trend	SL M
1865	2299	(2630)
1870	3122	(3320)
1875	4132	(4300)
1880	5301	5350
1885	6569	6400
1890	7850	7500
1895	9054	8750
1900	10112	10000
1905	10988	11200
1910	11678	11800
1915	12201	12100
1920	12585	12500
1925	12862	12800

Kuznets' equation:

$$y = \frac{13498}{1+10^{(0.52152-0.16611x)}}$$

x in units of five years; origin at 1870.

IN the analysis of the cotton production series, no correction for curvature was made. The agreement between Kuznets' trend line and SL M is close. The difference is greatest, although not very great, in the period 1865 to 1875 in which, however, the values of SL M are admittedly only tentative. In that early section, SL M is slightly higher than the trend line.

THE major cycles obtained by the two methods are quite unlike. SL B follows the data closely, possibly too closely; for example, the little peak in SL B in 1897 and 1898 might have been reduced, but probably the similar peaks in 1889-90 and 1904-05 should not be smoothed away entirely. Kuznets' smoothing is extreme. It reduces much of the amplitude of movement from the major cycle and increases it in the short business cycle. Probably Kuznets' line which corresponds to SL B does not follow the data closely enough.

COTTON PRODUCTION.

The values of the standard deviation, by the several calculations:

	From Kuznets' figures, based on period 1866 to 1915	Figures secured by the method of smoothing by stages	
		based on period 1866 to 1924	based on period 1866 to 1939
The short business cycle	15.9%	10.5%	10.9%
Years included (omit terminal half-cycles)	full	1868 to 1922	1868 to 1936
The major cycle	7.6%	7.6%	9.2%
Years included (omit terminal half-cycles)	full	1880 to 1917	1880 to 1929

Table K. COTTON PRODUCTION IN THE UNITED STATES, 1866 TO 1939

Two Stages of Smoothing. Columns numbered as in Tables B and C.

Source: Yearbooks of the United States Department of Agriculture.

Part a (four pages)

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a	11b	11c	12a	12b	13	24	1
Year	Cotton Production thousands bales	Cotton Production logarithm	Phase Point (short cycle)	Cycle	Yearly Figures Included in the Cycle Begin	Yearly Figures Included in the Cycle Middle	Yearly Figures Included in the Cycle End	Length in Years	Moving Total of Logs	Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Cotton Production (thousands bales)	Smoothing Line B thousands bales	Smoothing Line B logarithm	Phase Point (major cycle)	SL M (thousands bales)	Year
1866	1750	3.24340										(2000)	3.30103		(2600)	1866
67	2340	3.36922	r_1 p_1	$r_{1,2}$	1867	1867-68	1868	2	6.74580	3.37290	2359	(2200)	3.34242		(2730)	67
68	2380	3.37658	f_1 t_1 r_2	$p_{1,2}$	1867	1868	1870	4	13.80444	3.45111	2826	2430	3.38561		(2900)	68
69	3012	3.47886		$f_{1,2}$ $t_{1,2}$	1868 1868	1869 1869-70	1870 1871	3 4	10.43522 13.84227	3.47841 3.46057	3009 2888	2670	3.42651		(3030)	69
1870	3800	3.57978	p_2 f_2	$r_{2,3}$	1869	1870	1871	3	10.46569	3.48856	3080	2930	3.46687		(3200)	1870
71	2553	3.40705	t_2	$p_{2,3}$ $f_{2,3}$	1870 1871	1871 1871-72	1872 1872	3 2	10.58012 6.98034	3.52671 3.49017	3363 3092	3210	3.50650		(3400)	71
72	3920	3.59329	r_3 p_3	$t_{2,3}$	1871	1872	1873	3	10.56654	3.52218	3328	3500	3.54407	r_1	(3600)	72
73	3683	3.56620	f_3 t_3	$r_{3,4}$ $p_{3,4}$	1872 1872	1873 1873-74	1874 1875	3 4	10.63510 14.34462	3.54837 3.58613	3535 3856	3800	3.57978		(3810)	73
74	3941	3.59561	r_4	$f_{3,4}$	1873	1874	1875	3	10.75133	3.58378	3835	4080	3.61066		(4000)	74
75	5123	3.70952	p_4	$t_{3,4}$	1874	1875	1876	3	10.95232	3.65077	4475	4370	3.64048		(4200)	75
76	4438	3.64719	f_4	$r_{4,5}$	1875	1876	1878	4	14.71685	3.67921	4778	4630	3.66558		(4400)	76
77	4370	3.64048	t_4									4900	3.69020		(4650)	77
78	5244	3.71066	r_5	$p_{4,5}$ $f_{4,5}$	1875 1876	1877-78 1878	1879 1880	5 5	18.47689 18.56966	3.69538 3.71393	4959 5175	5200	3.71600		(4880)	78
79	5755	3.76044		$t_{4,5}$	1877	1879	1881	5	18.65934	3.73187	5294	5500	3.74036		(5150)	79
1880	6343	3.80229	p_5 f_5	$r_{5,6}$	1878	1880	1881	4	15.01886	3.75472	5685	5800	3.76343		5350	1880
81	5456	3.73687	t_5	$p_{5,6}$	1880	1881	1882	3	11.38158	3.79389	6221	6060	3.78247	p_1	5600	81

Table K (continued) Cotton Production in the United States

1 Year	2a or 4a Cotton Production thousands bales	2b or 4b Cotton Production logarithm	5 Phase Point (short cycle)	12a Smoothing Line B thousands bales	12b Smoothing Line B logarithm	13 Phase Point (major cycle)	24 SL M (thousands bales)	1 Year
1882	6957	3.84242	r_6 p_6	6200	3.79239		5850	1882
83	5701	3.75595	f_6 t_6	6200	3.79239	f_1	6050	83
84	5682	3.75450		6200	3.79239		6300	84
85	6575	3.81790	r_7 p_7	6350	3.80277	t_1	6500	85
86	6446	3.80929	f_7 t_7	6600	3.81954		6700	86
87	7020	3.84634	r_8 p_8 f_8	6850	3.83569		6950	87
88	6941	3.84142		7200	3.85733	r_2	7150	88
89	7473	3.87350	t_8 r_9	7650	3.88366		7350	89
1890	8674	3.93822		7850	3.89487	p_2	7650	1890
91	9018	3.95511	p_9	7900	3.89763	f_2	7900	91
92	6664	3.82373	f_9 t_9	7800	3.89210		8150	92
93	7493	3.87466	r_{10}	7800	3.89210		8400	93
94	9476	3.97662	p_{10}	7950	3.90037	t_2	8650	94
95	7161	3.85497	f_{10} t_{10}	8350	3.92169		8900	95
96	8533	3.93110	r_{11}	9200	3.96379	r_3	9150	96
97	10898	4.07535		10000	4.00000		9400	97
98	11189	4.04879	p_{11} f_{11}	10300	4.01284	p_3	9700	98
99	9345	3.97058	t_{11}	10100	4.00432		10000	99
1900	10123	4.00531	r_{12} p_{12}	9700	3.98677	f_3	10300	1900
01	9510	3.97818	f_{12} t_{12}	9850	3.99354	t_3	10600	01

6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Yearly Figures Included in the Cycle Middle	9 Yearly Figures Included in the Cycle End	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Cotton Production (thousands bales)
$f_{5,6}$	1881	1881-82	1882	2	7.57929	3.78964	6161
$t_{5,6}$	1881	1882	1883	3	11.33524	3.77841	6004
$r_{6,7}$	1882	1883	1884	3	11.35287	3.78429	6085
$p_{6,7}$	1882	1883-84	1885	4	15.17077	3.79269	6204
$f_{6,7}$	1883	1884	1885	3	11.32835	3.77612	5972
$t_{6,7}$	1884	1885	1886	3	11.38169	3.79390	6222
$r_{7,8}$	1885	1885-86	1886	2	7.62719	3.81359	6510
$p_{7,8}$	1885	1886	1887	3	11.47353	3.82451	6676
$f_{7,8}$	1886	1886-87	1887	2	7.65563	3.82781	6727
$t_{7,8}$	1886	1887	1888	3	11.49705	3.83235	6798
$r_{8,9}$	1887	1888	1889	3	11.56126	3.85375	7141
$p_{8,9}$	1887	1889	1890	4	15.49948	3.87487	7497
$f_{8,9}$	1888	1889-90	1891	4	15.60825	3.90206	7981
$t_{8,9}$	1889	1890	1892	4	15.59056	3.89764	7900
$r_{9,10}$	1890	1891	1893	4	15.59172	3.89793	7906
$p_{9,10}$	1891	1892	1894	4	15.63012	3.90753	8082
$f_{9,10}$	1892	1893	1894	3	11.67501	3.89167	7792
$t_{9,10}$	1893	1894	1895	3	11.70625	3.90208	7981
$r_{10,11}$	1894	1895	1896	3	11.76269	3.92090	8335
$p_{10,11}$	1894	1896	1897	4	15.80004	3.95001	8913
$f_{10,11}$	1895	1897	1898	4	15.87221	3.96805	9291
$t_{10,11}$	1896	1897	1899	4	15.98782	3.99696	9930
$r_{11,12}$	1897	1898	1899	3	12.05672	4.01891	10688
$p_{11,12}$	1898	1899	1900	3	12.02468	4.00823	10191
$f_{11,12}$	1899	1900	1900	2	7.97589	3.98794	9726
$t_{11,12}$	1899	1900	1901	3	11.95407	3.98469	9654
$r_{12,13}$	1900	1900-01	1901	2	7.98349	3.99174	9812
$p_{12,13}$	1900	1901	1902	3-	12.01006	4 00252	10058

(Table K, part a, is continued on next page)

Table K, Part a (continued) Cotton Production, two stages of smoothing

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a	11b	11c	12a	12b	13	24	1
Year	Cotton Production		Phase Point	Cycle	Years Included			Length in Years	Moving Total of Logs	Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Cotton Production (thousands bales)	Smoothing Line B		Phase Point	SL M (thousands bales)	Year
	thousands bales	logarithm			Begin	Middle	End					thousands bales	logarithm			
			r_{13}	$f_{12,13}$	1901	1901-02	1902	2	8.00475	4.00238	10055					
1902	10631	4.02657	p_{13}	$t_{12,13}$	1901	1902	1903	3	11.99823	3.99941	9986	10200	4.00860		10800	1902
			f_{13}	$r_{13,14}$	1902	1902	1903	2	8.02005	4.01002	10233					
03	9851	3.99348	t_{13}	$p_{13,14}$	1902	1903	1904	3	12.14839	4.04946	11206	10800	4.03342	r_4	11200	03
			r_{14}	$f_{13,14}$	1903	1903-04	1904	2	8.12182	4.06091	11506					
04	13438	4.12834	p_{14}	$t_{13,14}$	1903	1904	1905	3	12.14610	4.04870	11187	11300	4.05308		11300	04
			f_{14}	$r_{14,15}$	1904	1904-05	1905	2	8.15262	4.07631	11921					
05	10575	4.02428	t_{14}	$p_{14,15}$	1904	1905	1906	3	12.27562	4.09187	12356	11700	4.06819		11400	05
			r_{15}	$f_{14,15}$	1905	1905-06	1906	2	8.14728	4.07364	11848					
06	13274	4.12300	p_{15}	$t_{14,15}$	1905	1906	1907	3	12.19287	4.06429	11595	11900	4.07555		11600	06
			f_{15}	$r_{15,16}$	1906	1906-07	1907	2	8.16860	4.08430	12142					
07	11107	4.04560	t_{15}	$p_{15,16}$	1906	1907	1908	3	12.29055	4.09685	12498	12000	4.07918		11700	07
			r_{16}	$f_{15,16}$	1907	1907-08	1908	2	8.16755	4.08377	12127					
08	13242	4.12195	p_{16}	$t_{15,16}$	1907	1908	1909	3	12.16777	4.05592	11374	11800	4.07188		11750	08
			f_{16}													
09	10005	4.00022	t_{16}	$r_{16,17}$	1908	1909	1910	3	12.18697	4.06232	11543	11800	4.07188		11800	09
				$p_{16,17}$	1908	1909-10	1911	4	16.38268	4.09567	12464					
1910	11609	4.06480	r_{17}	$f_{16,17}$	1909	1910	1911	3	12.26072	4.08691	12216	12500	4.09691		11900	1910
11	15693	4.19571	p_{17}	$t_{16,17}$	1910	1911	1912	3	12.39734	4.13245	13566	13900	4.14302		11950	11
12	13703	4.13682	f_{17}	$r_{17,18}$	1911	1912	1913	3	12.48347	4.16116	14493	14700	4.16732	p_4	12000	12
			t_{17}	$p_{17,18}$	1911	1912-13	1914	4	16.69125	4.17256	14879					
13	14156	4.15094	r_{18}	$f_{17,18}$	1912	1913	1914	3	12.49553	4.16518	14628	14800	4.17026		12100	13
14	16135	4.20777	p_{18}	$t_{17,18}$	1913	1914	1916	4	16.46641	4.11660	13080	14200	4.15229		12150	14
15	11192	4.04891	f_{18}									13000	4.11394		12200	15
				$r_{18,19}$	1914	1915-16	1917	4	16.36865	4.09216	12364			f_4		
16	11450	4.05881	t_{18}	$p_{18,19}$	1914	1916	1918	5	20.44931	4.08936	12299	12200	4.08636		12250	16
				$f_{18,19}$	1915	1916-17	1918	4	16.24154	4.06038	11492					
17	11302	4.05316										11700	4.06819		12300	17
			r_{19}	$t_{18,19}$	1916	1917-18	1919	4	16.25033	4.06258	11550					
1918	12041	4.08066	p_{19}									11700	4.06819		12400	1918

Table K, Part a (continued) Cotton Production, two stages of smoothing

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a	11b	11c	12a	12b	13	24	1
Year	Cotton Production		Phase Point	Cycle	Years Included			Length in Years	Moving Total of Logs	Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Cotton Production (thousands bales)	Smoothing Line B		Phase Point	SL M (thousands bales)	Year
	thousands bales	logarithm			Begin	Middle	End					thousands bales	logarithm			
			f_{19}	$r_{19,20}$	1918	1918-19	1919	2	8.13836	4.06918	11726					
19	11421	4.05770	t_{19}	$p_{19,20}$	1918	1919	1920	3	12.26676	4.08892	12272	11800	4.07188		12450	19
			r_{20}	$f_{19,20}$	1919	1919-20	1920	2	8.18610	4.09305	12389					
1920	13440	4.12840	p_{20}	$t_{19,20}$	1919	1920	1921	3	12.08669	4.02890	10688	11000	4.04139		12510	1920
			f_{20}	$r_{20,21}$	1920	1920-21	1921	2	8.02899	4.01449	10339					
21	7954	3.90059	t_{20}	$p_{20,21}$	1920	1921	1922	3	12.01853	4.00618	10143	9300	3.96848		12600	21
			r_{21}	$f_{20,21}$	1921	1921-22	1922	2	7.89013	3.94506	8812			t_4		
22	9762	3.98954	p_{21}	$t_{20,21}$	1921	1922	1923	3	11.89616	3.96872	9305	9400	3.97313		12650	22
			f_{21}													
23	10140	4.00604	t_{21}	$r_{21,22}$	1922	1923	1923	2	7.99559	3.99779	9949	10700	4.02938		12700	23
			r_{22}													
24	13628	4.13443		$p_{21,22}$	1922	1924	1925	4	16.33697	4.08424	12141	13200	4.12057	r_5	12750	24
25	16105	4.20696		$f_{21,22}$	1923	1925	1926	4	16.60217	4.15054	14143	14600	4.16435		12800	25
				$t_{21,22}$	1923	1925	1927	5	20.71465	4.14293	14221					
26	17978	4.25474	p_{22}	$r_{22,23}$	1924	1926	1928	5	20.86930	4.17386	14923	14800	4.17026		12900	26
27	12956	4.11247	f_{22}	$p_{22,23}$	1926	1927	1928	3	12.52730	4.17577	14989	14600	4.16435		12950	27
			t_{22}													
28	14477	4.16068	r_{23}	$f_{22,23}$	1927	1928	1929	3	12.44414	4.14805	14062	14200	4.15229		13050	28
				$t_{22,23}$	1927	1928-29	1930	4	16.57101	4.14275	13891			p_5		
29	14825	4.17100	p_{23}	$r_{23,24}$	1928	1929	1930	3	12.45853	4.15188	14187	14200	4.15229		13100	29
			f_{23}													
1930	13392	4.12685	t_{23}	$p_{23,24}$	1929	1930	1931	3	12.53075	4.17692	15029	14800	4.17026		(13200)	1930
			r_{24}	$f_{23,24}$	1930	1930-31	1931	2	8.35915	4.17957	15121					
31	17096	4.23290	p_{24}	$t_{23,24}$	1930	1931	1932	3	12.47375	4.15792	14385	14700	4.16732		(13200)	31
			f_{24}	$r_{24,25}$	1931	1931-32	1932	2	8.34691	4.17345	14909					
32	13002	4.11401	t_{24}	$p_{24,25}$	1931	1932	1933	3	12.46241	4.15414	14261	13700	4.13672		(13200)	32
			r_{25}											f_5		
33	13047	4.11551	p_{25}	$f_{24,25}$	1932	1933	1933	2	8.22952	4.11476	13024	12300	4.08990		(13250)	33
			f_{25}	$t_{24,25}$	1932	1933	1934	3	12.21341	4.07114	11780					
34	9636	3.98390	t_{25}	$r_{25,26}$	1933	1934	1935	3	12.12626	4.04209	11018	11000	4.04139	t_5	(13300)	34
35	10638	4.02686		$p_{25,26}$	1933	1935	1937	5	20.49720	4.09944	12573	11100	4.04532	r_6	(13300)	35
				$f_{25,26}$	1934	1935-36	1937	4	16.38167	4.09042	12315					
36	12399	4.09339	r_{26}									12500	4.09691		(13300)	33
				$t_{25,26}$	1935	1936-37	1939	5	20.54740	4.10948	12867					
37	18946	4.27752	p_{26}									(13700)	4.13672		(13300)	37
38	11943	4.07712	f_{26}									(14200)	4.12752		(13300)	38
1939	11817	4.07251	t_{26}									(14400)	4.15836		(13250)	1939

Table K. COTTON PRODUCTION

Part b Part of the calculations for the second stage of smoothing.

Columns numbered as in Tables B and C.

14 Cycle	15	16 Yearly Figures Included in the Cycle	17	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (thousands bales)
	Begin	Middle	End				
$r_{1,2}$	1872	1880	1888	17	63.42553	3.73091	5382
$p_{1,2}$	1881	1885	1889	9	34.35863	3.81763	6571
$f_{1,2}$	1884	1888	1891	8	30.77388	3.84877	7059
$t_{1,2}$	1886	1889-90	1893	8	30.97292	3.87162	7441
$r_{2,3}$	1888	1892	1895	8	31.13975	3.89247	7807
$p_{2,3}$	1890	1893-94	1897	8	31.36255	3.92032	8324
$f_{2,3}$	1891	1895	1899	9	35.48484	3.94276	8765
$t_{2,3}$	1894	1897-98	1901	8	31.78322	3.97290	9395
$r_{3,4}$	1896	1899-00	1903	8	32.00318	4.00040	10009
$p_{3,4}$	1898	1905	1912	15	60.86640	4.05776	11422
$f_{3,4}$	1900	1907-08	1915	16	65.28573	4.08036	12033
$t_{3,4}$	1902	1911	1921	20	81.61001	4.08050	12037
$r_{4,5}$	1903	1913-14	1921	19	77.60141	4.08428	12142
$p_{4,5}$	1913	1920	1928	16	65.51531	4.09471	12437
$f_{4,5}$	1916	1924	1932	17	69.70541	4.10032	12599
$t_{4,5}$	1922	1928	1934	13	53.53221	4.11786	13118
$r_{5,6}$	1924	1930	1936	13	53.67193	4.12861	13446

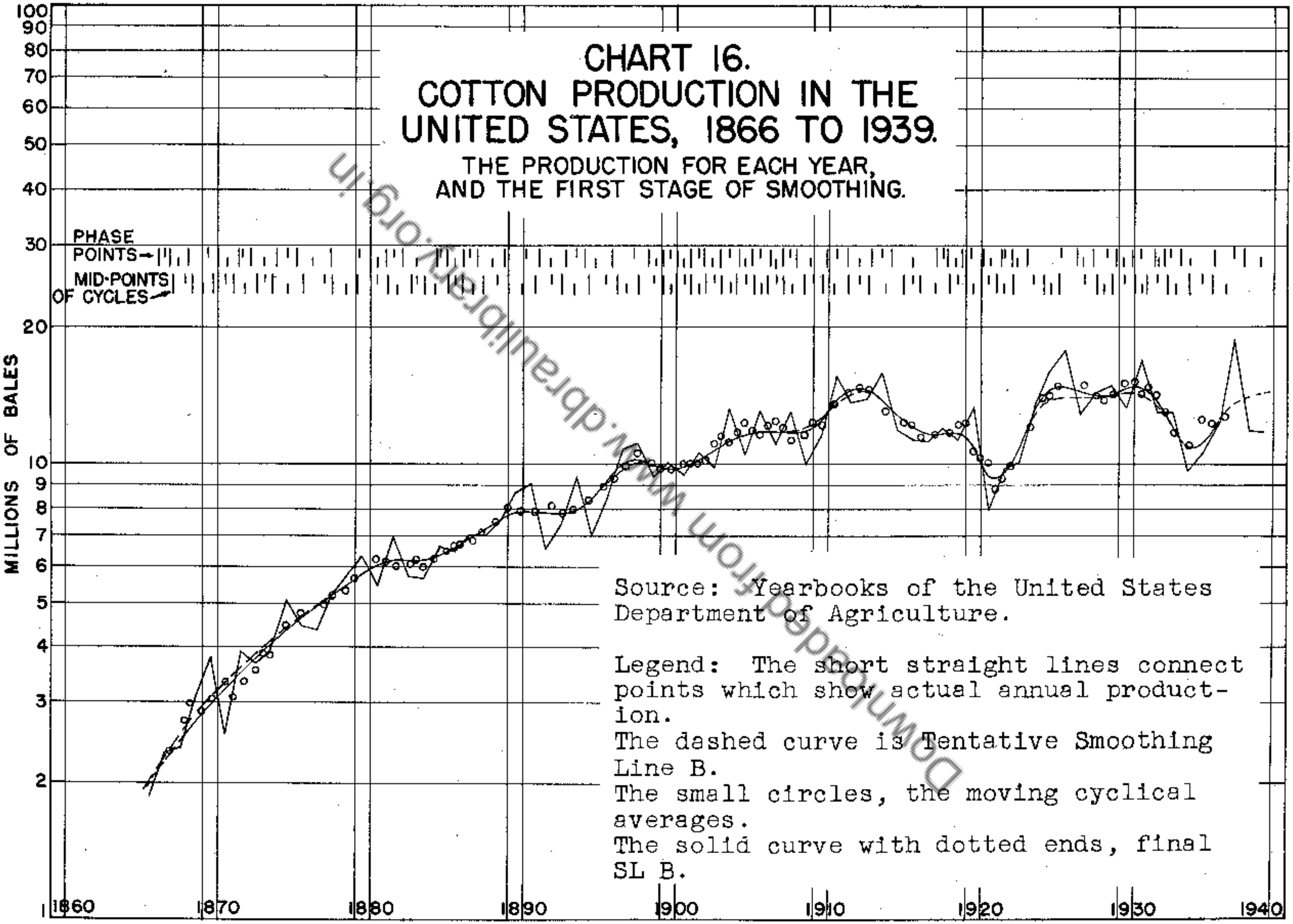


CHART 16.
COTTON PRODUCTION IN THE UNITED STATES, 1866 TO 1939.
THE PRODUCTION FOR EACH YEAR, AND THE FIRST STAGE OF SMOOTHING.

Source: Yearbooks of the United States Department of Agriculture.

Legend: The short straight lines connect points which show actual annual production.
The dashed curve is Tentative Smoothing Line B.
The small circles, the moving cyclical averages.
The solid curve with dotted ends, final SL B.

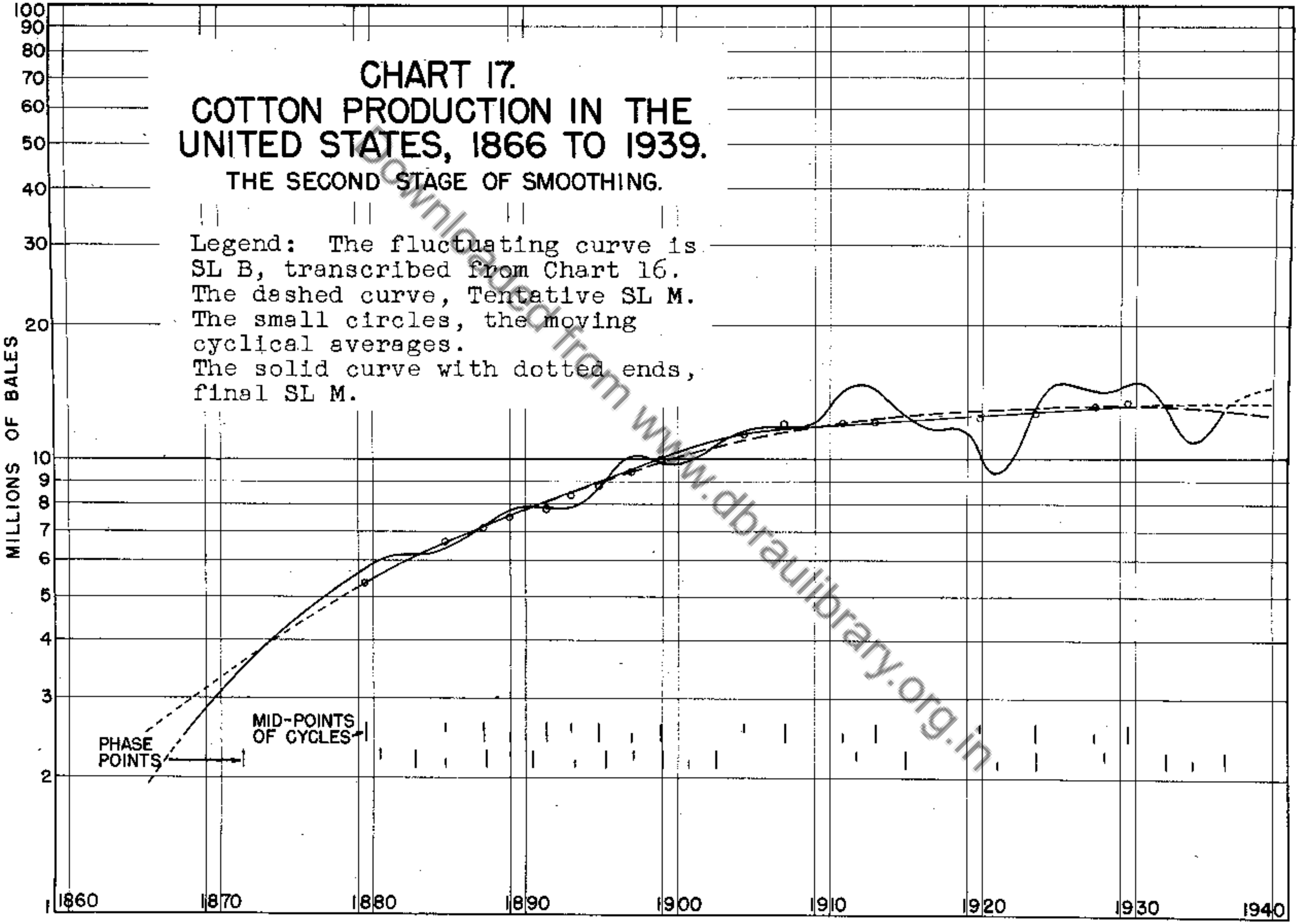


CHART 17.

COTTON PRODUCTION IN THE UNITED STATES, 1866 TO 1939.

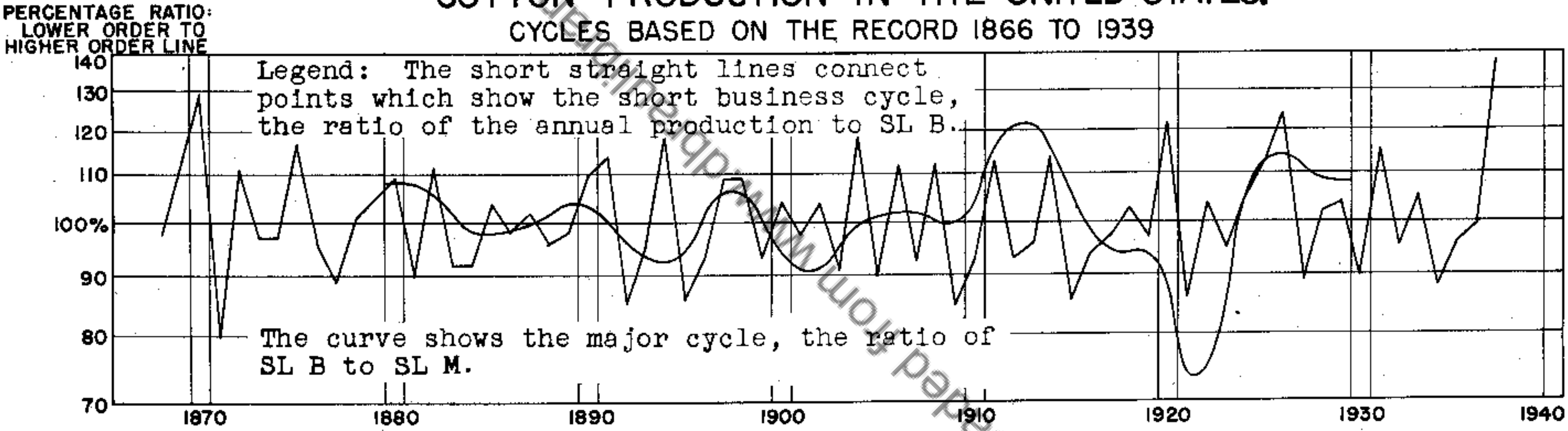
THE SECOND STAGE OF SMOOTHING.

Legend: The fluctuating curve is SL B, transcribed from Chart 16.
The dashed curve, Tentative SL M.
The small circles, the moving cyclical averages.
The solid curve with dotted ends, final SL M.

CHART 18.

COTTON PRODUCTION IN THE UNITED STATES.

CYCLES BASED ON THE RECORD 1866 TO 1939



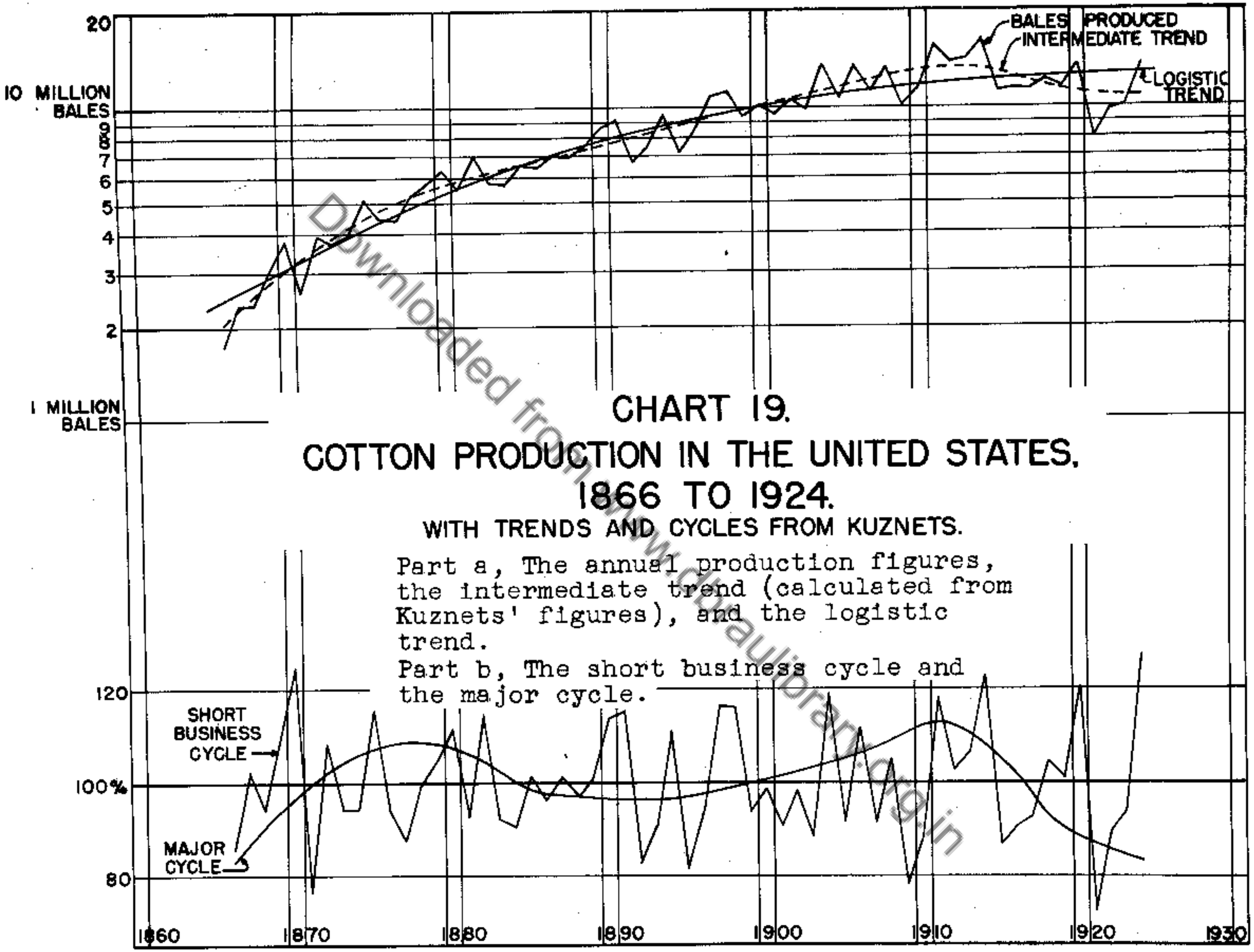


CHART 19.

COTTON PRODUCTION IN THE UNITED STATES. 1866 TO 1924.

WITH TRENDS AND CYCLES FROM KUZNETS.

Part a, The annual production figures, the intermediate trend (calculated from Kuznets' figures), and the logistic trend.

Part b, The short business cycle and the major cycle.

Table L. COTTON PRODUCTION IN THE UNITED STATES

Calculation of standard measures of the two orders of cycles.

Based on the record 1866 to 1939. (Three pages)

Columns numbered as in Tables D and E.

1 Year	2 (or 4)a Cotton Production	12 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of bales)			The short business cycle			The major cycle			
1866	1750									1866
67	2340									67
68	2380	2430		98	- 2	4				68
69	3012	2670		113	+13	169				69
1870	3800	2930		130	+30	900				1870
71	2553	3210		80	-20	400				71
72	3920	3500		112	+12	144				72
73	3683	3800		97	- 3	9				73
74	3941	4080		97	- 3	9				74
75	5123	4370		117	+17	289				75
76	4438	4630		96	- 4	16				76
77	4370	4900		89	-11	121				77
78	5244	5200		101	+ 1	1				78
79	5755	5500		105	+ 5	25				79
1880	6343	5800	5350	109	+ 9	81	108	+ 8	64	1880
81	5456	6060	5600	90	-10	100	108	+ 8	64	81
82	6957	6200	5850	112	+12	144	106	+ 6	36	82
83	5701	6200	6050	92	- 8	64	102	+ 2	4	83
84	5682	6200	6300	92	- 8	64	98	- 2	4	84
85	6575	6350	6500	104	+ 4	16	98	- 2	4	85
86	6446	6600	6700	98	- 2	4	99	- 1	1	86
87	7020	6850	6950	102	+ 2	4	99	- 1	1	87
88	6941	7200	7150	96	- 4	16	101	+ 1	1	88
89	7473	7650	7350	98	- 2	4	104	+ 4	16	89
1890	8674	7850	7650	110	+10	100	103	+ 3	9	1890
91	9018	7900	7900	114	+14	196	100	0	0	91
92	6664	7800	8150	85	-15	225	96	- 4	16	92
93	7493	7800	8400	96	- 4	16	93	- 7	49	93
94	9476	7950	8650	119	+19	361	92	- 8	64	94
95	7161	8350	8900	86	-14	196	94	- 6	36	95
96	8533	9200	9150	93	- 7	49	101	+ 1	1	96
97	10898	10000	9400	109	+ 9	81	106	+ 6	36	97
98	11189	10300	9700	109	+ 9	81	106	+ 6	36	98
1899	9345	10100	10000	93	- 7	49	101	+ 1	1	1899

Table L (continued) Cotton Production in the United States

1 Year	2 (or 4)a Cotton Production	12 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of bales)			The short business cycle			The major cycle			
1900	10123	9700	10300	104	+ 4	16	94	- 6	36	1900
01	9510	9850	10600	97	- 3	9	91	- 9	81	01
02	10631	10200	10800	104	+ 4	16	91	- 9	81	02
03	9851	10800	11200	91	- 9	81	96	- 4	16	03
04	13438	11300	11300	119	+19	361	99	- 1	1	04
05	10575	11700	11400	90	-10	100	101	+ 1	1	05
06	13274	11900	11600	112	+12	144	102	+ 2	4	06
07	11107	12000	11700	93	- 7	49	102	+ 2	4	07
08	13242	11800	11750	112	+12	144	100	0	0	08
09	10005	11800	11800	85	-15	225	99	- 1	1	09
1910	11609	12500	11900	93	- 7	49	105	+ 5	25	1910
11	15693	13900	11950	113	+13	169	116	+16	256	11
12	13703	14700	12000	93	- 7	49	121	+21	441	12
13	14156	14800	12100	96	- 4	16	122	+22	484	13
14	16135	14200	12150	114	+14	196	116	+16	256	14
15	11192	13000	12200	86	-14	196	106	+ 6	36	15
16	11450	12200	12250	94	- 6	36	99	- 1	1	16
17	11302	11700	12300	97	- 3	9	94	- 6	36 end of the short period for calculation of sd.	17
18	12041	11700	12400	103	+ 3	9	94	- 6	36	18
19	11421	11800	12450	97	- 3	9	94	- 6	36	19
1920	13440	11000	12510	122	+22	484	87	-13	169	1920
21	7954	9300	12600	86	-14	196	74	-26	676	21
22	9762	9400	12650	104	+ 4	16 end of the short period for calculation of sd.	74	-26	676	22
23	10140	10700	12700	95	- 5	25	84	-16	256	23
24	13628	13200	12750	103	+ 3	9	103	+ 3	9	24
25	16105	14600	12800	110	+10	100	113	+13	169	25
26	17978	14800	12900	121	+21	441	114	+14	196	26
27	12956	14600	12950	89	-11	121	112	+12	144	27
28	14477	14200	13050	102	+ 2	4	108	+ 8	64	28
29	14825	14200	13100	104	+ 4	16	108	+ 8	64	29
1930	13392	14800		90	-10	100			4698	1930
31	17096	14700		116	+16	256				31
32	13002	13700		95	- 5	25				32
33	13047	12300		106	+ 6	36				33
34	9636	11000		88	-12	144				34
35	10638	11100		96	- 4	16				35
36	12399	12500		99	- 1	1				36
37	18946									37
38	11943									38
1939	11817									1939

$\Sigma(d^2) = 7798$

sd $= \sqrt{\frac{4698}{50}} = 9.7\%$, in the major cycle (the full period)

sd $= \sqrt{\frac{7798}{69}} = 10.6\%$, in the short business cycle. (the full period)

(Table L is concluded on the next page.)

Table L (concluded) Cotton Production in the United States

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles

28
of Actual about SL B (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1	-	-	.6	.6
2	1.3	.7	.6	.7
3	.3	.5	.8	.8
4	.7	.6	1.0	1.1
5	1.9	.7	.5	.5
6	.5	.6	.9	1.3
7	.3	.4	.5	.5
8	.5	.5	1.0	.7
9	1.5	.8	.7	1.0
10	.7	.5	.7	1.1
11	1.5	.9	.5	.5
12	.5	.4	.5	.5
13	.6	.3	.5	.5
14	.6	.7	.3	.5
15	.5	.7	.3	.5
16	.5	.3	.9	1.1
17	.7	.7	.8	.8
18	.7	.7	1.3	1.6
19	.5	.3	.5	.4
20	.7	.7	.5	.8
21	.2	.5	.3	.7
22	2.3	1.0	.4	.7
23	.9	.6	.5	.5
24	.7	.9	.2	.4
25	.4	.6	.7	1.6
26	1.3	-	-	-
Average length	.8 yr.	.6 yr.	.6 yr.	.8 yr.

Total length of typical short business cycle 2.8 yrs.

33
of SL B about SL M (the major cycle)

Cycle Number	rp	pf	ft	tr
1	-	-	2.0	2.7
2	1.6	1.4	2.7	2.0
3	1.8	1.9	1.7	1.7
4	9.2	3.3	6.0	2.5
5	4.5	4.0	-	-
Average Length	4.3	2.7	3.1	2.2

Total length of typical major cycle 12.3 yrs.

Percentage Deviations at Peaks and Troughs

29
of actual from SL B (the short business cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1868	-2%
1870	+30%	71	20
72	12	73	3
75	17	77	11
80	9	81	10
82	12	84	8
85	4	86	2
87	2	88	4
91	14	92	15
94	19	95	14
98	9	99	7
1900	4	1901	3
02	4	03	9
04	19	05	10
06	12	07	7
08	10	09	18
11	15	12	5
14	17	16	13
18	3	19	3
20	22	21	14
22	14	23	5
26	24	27	11
29	4	30	10
31	16	32	5
1936	6	1934	12
Average Deviation in the short business cycle	12.4% at peak		8.8% at trough

34
of SL B from SL M (the major cycle)

Year	Deviation	Year	Deviation
1881	+ 8%	1885	-1%
1889	5	1894	6
1897	8	1901	4
1912	21	1921	26
1926	12		
Average Deviation in the major cycle	10.8% at peak		7.4% at trough

Section 4. CRUDE PETROLEUM OUTPUT.

COMPARISON of Kuznets' trend line with the smoothing line SL M (thousands of barrels)

	Kuznets' Trend	SL M
1860	2375	(1400)
1865	3719	(2750)
1870	5821	(5400)
1875	9111	10400
1880	14256	18600
1885	22293	28000
1890	34837	40000
1895	54380	56000
1900	84718	80000
1905	131618	120000
1910	203602	185000
1915	312870	298000
1920	476051	450000
1925	713709	670000

Kuznets' equation:

$$y = \frac{6,060,606}{1+10^{(3.01705-0.19477x)}}$$

x in units of 5 years; origin at 1870.

SL M fits rather closely to the data; so closely, in fact, that it still exhibits two cycles. Consequently the amplitude about it should be expected to be less than about the trend line of Kuznets. SL M falls below the trend line in 1860 to 1865, rises above it in 1880 to 1890, and falls below it again after 1900. After 1919, SL M begins to be pulled down by the subsequent decline in production. Kuznets' line is almost straight on the semi-logarithmic chart, almost of the type $y = k^x$. He has arrived at his secondary smoothing line, which separates his two orders of cycles, by using an eleven-year average prior to 1902 and a five-year average thereafter. This procedure, clearly, is due to the inadequacy of a fixed length moving average. The contour of the major cycles gotten by the two methods is quite different. SL M, which was shaped to achieve running equality of areas intercepted, managed thereby to secure a reasonably regular major cycle, but did it at the expense of retaining cycles in SL M itself. Kuznets, on the other hand, made his trend line into practically a straight line on the semi-logarithmic paper, and, as a result, he got a major cycle which is rather a jumble. Possibly this series on the output of crude petroleum best illustrates the shortcomings of both methods. In defense of the method of smoothing by stages, it may be emphasized that if the operator is unwilling to accept as his secular trend, smoothing line M, containing two cycles in the eighty years, he is at liberty to undertake another stage of smoothing.

CRUDE PETROLEUM OUTPUT.

The values of the standard deviation, by the several calculations:

	From Kuznets' figures, based on period 1854 to 1924	Figures secured by the method of smoothing by stages	
		based on period 1861 to 1924	based on period 1861 to 1938
The short business cycle	13.0%	9.0%	8.4%
Years included (omit terminal half-cycle)	full	1864 to 1922	1863 to 1936
The major cycle	16.0%	12.7%	12.9%
Years included (omit terminal half-cycle)	full	1875 to 1915	1875 to 1925

Table M. CRUDE PETROLEUM OUTPUT IN THE UNITED STATES, 1861 TO 1938

Two stages of smoothing. Columns numbered as in Tables B and C.

Sources: Mineral Resources of the United States; and Yearbooks of Commerce.

Part a (four pages)

1 Year	2a or 4a Crude Petroleum Output thousands barrels	2b or 4b Crude Petroleum Output Logarithm	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the cycle Begin	8 Yearly Figures Included in the cycle Middle	9 Yearly Figures Included in the cycle End	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Crude Petroleum Output (thousands barrels)	12a Smoothing Line B thousands barrels	12b Smoothing Line B Logarithm	13 Phase Point (major cycle)	24 SL M (thousands barrels)	1 Year
1861	2114	3.32510	r_1									(1950)	3.29003		(1620)	1861
62	3057	3.48530	p_1									(2140)	3.33041		(1850)	62
63	2611	3.41681	f_1	$r_{1,2}$	1861	1863	1865	5	16.95032	3.39006	2455	(2350)	3.37107		(2100)	63
64	2116	3.32552	t_1	$p_{1,2}$	1862	1864	1866	5	17.18120	3.43624	2731	2600	3.41497	f_1	(2400)	64
65	2498	3.39759	r_2	$f_{1,2}$	1864	1865	1866	3	10.27917	3.42639	2669	2880	3.45939		(2750)	65
66	3598	3.55606	p_2	$t_{1,2}$	1865	1866	1868	4	14.04013	3.50803	3221	3170	3.50106		(3150)	66
67	3347	3.52466	f_2	$r_{2,3}$	1866	1867	1869	4	14.26734	3.56683	3688	3500	3.54407		(3600)	67
68	3646	3.56182	t_2	$p_{2,3}$	1866	1868	1870	5	17.98841	3.59768	3960	3900	3.59106		(4100)	68
69	4215	3.62480	r_3	$f_{2,3}$	1867	1869	1870	4	14.43235	3.60809	4056	4350	3.63849		(4700)	69
1870	5261	3.72107	p_3 f_3	$t_{2,3}$	1868	1870	1871	4	14.62411	3.65603	4529	4900	3.69020		(5400)	1870
71	5205	3.71642	t_3	$r_{3,4}$	1870	1871	1872	3	11.23635	3.74545	5565	5650	3.75205	t_1	(6100)	71
72	6293	3.79886	r_4	$p_{3,4}$ $f_{3,4}$	1870 1871	1872 1872-73	1873 1874	4 4	15.23172 15.54915	3.80793 3.88729	6426 7714	6700	3.82607		(7000)	72
73	9894	3.99537		$t_{3,4}$	1872	1873-74	1875	4	15.77662	3.94415	8793	7900	3.89763		(8000)	73
74	10927	4.03850	p_4									9100	3.95904		(9100)	74
1875	8788	3.94389	f_4									10400	4.01703		10400	1875

Table M, part a. (continued) Crude Petroleum Output.

1 Year	2a or 4a Crude Petroleum Output thousands barrels	2b or 4b Logarithm	5 Phase Point (short cycle)	12a Smoothing Line B thousands barrels	12b Logarithm	13 Phase Point (major cycle)	24 SL M (thousands barrels)	1 Year
1876	9133	3.96061	t_4	11700	4.06819		11800	1876
77	13350	4.12548		13200	4.12057		13300	77
78	15397	4.18744	r_5	15000	4.17609		15200	78
79	19914	4.20178		19000	4.27875	r_1	17000	79
1880	26286	4.41972	p_5	25500	4.40654		18600	1880
81	27661	4.44187	f_5 t_5	28500	4.45484	p_1	20200	81
82	30350	4.48216	r_6 p_6	28000	4.44716		22000	82
83	23450	4.37014	f_6 t_6	25500	4.40654	f_2	24000	83
84	24218	4.38414	r_7 p_7	23500	4.37107		26000	84
85	21859	4.33963	f_7 t_7	23800	4.37658	t_2	28000	85
86	28065	4.44817	r_8 p_8	25800	4.41162		31000	86
87	28283	4.45153	f_8	28500	4.45484		33000	87
88	27612	4.44110	t_8	32000	4.50515		35000	88
89	35164	4.54610	r_9	37000	4.56820	r_2	37500	89
1890	45824	4.66109		41500	4.61805		40000	1890
91	54293	4.73474	p_9	46500	4.66745		43000	91
92	50515	4.70342	f_9	50500	4.70329	p_2	46000	92
93	48431	4.68512		53000	4.72428		50000	93
94	49345	4.69324	t_9	54000	4.73239		53000	94
95	52892	4.72339	r_{10}	54500	4.73640	f_3	56000	95
96	60960	4.78504	p_{10}	55000	4.74036		60000	96
1897	60476	4.78158	f_{10}	57500	4.75967		64000	1897

6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Middle	9 End	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Crude Petroleum Output (thousands barrels)
$r_{4,5}$	1873	1875	1878	6	24.25139	4.04188	11010
$p_{4,5}$	1874	1877	1880	7	28.87742	4.12535	13350
$f_{4,5}$	1875	1877-78	1880	6	24.83892	4.13982	13800
$t_{4,5}$	1876	1878	1880	5	20.89503	4.17901	15100
$r_{5,6}$	1878	1880	1881	4	17.25081	4.31270	20540
$p_{5,6}$	1880	1881	1882	3	13.34375	4.44792	28050
$f_{5,6}$	1881	1881-82	1882	2	8.92403	4.46202	28980
$t_{5,6}$	1881	1882	1883	3	13.29417	4.43139	27000
$r_{6,7}$	1882	1882-83	1883	2	8.85230	4.42615	26680
$p_{6,7}$	1882	1883	1884	3	13.23644	4.41215	25830
$f_{6,7}$	1883	1883	1884	2	8.75428	4.37714	23830
$t_{6,7}$	1883	1884	1885	3	13.09391	4.36464	23150
$r_{7,8}$	1884	1884-85	1885	2	8.72377	4.36189	23010
$p_{7,8}$	1884	1885	1886	3	13.17194	4.39065	24580
$f_{7,8}$	1885	1886	1887	3	13.23933	4.41311	25890
$t_{7,8}$	1885	1886-87	1888	4	17.68043	4.42011	26310
$r_{8,9}$	1886	1887	1889	4	17.88690	4.47173	29630
$p_{8,9}$	1887	1888-89	1890	4	18.09981	4.52495	33490
$f_{8,9}$	1888	1890	1892	5	23.08645	4.61929	41620
$t_{8,9}$	1888	1891	1894	7	32.46481	4.63912	43570
$r_{9,10}$	1889	1892	1895	7	32.74710	4.69101	49090
$p_{9,10}$	1891	1893-94	1896	6	28.32495	.72082	52580
$f_{9,10}$	1892	1895	1897	6	28.37179	4.72863	53530
$t_{9,10}$	1894	1897	1899	6	28.48290	4.74715	55870

(Table M, part e, is continued on next page)

Table M, Part a. (continued) Crude Petroleum Output, two stages of smoothing

1 Year	2a or 4a Crude Petroleum Output thousands barrels	2b or 4b Crude Petroleum Output Logarithm	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the cycle Begin	8 Yearly Figures Included in the cycle Middle	9 Yearly Figures Included in the cycle End	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Petroleum Output thousands barrels	12a Smoothing Line B thousands barrels	12b Smoothing Line B logarithm	13 Phase Point (major cycle)	24 SL M (thousands barrels)	1 Year
98	55364	4.74323		$r_{10,11}$	1896	1898	1901	6	28.71116	4.78519	60980	60500	4.78176		69000	98
99	57071	4.75642										65000	4.81291	t_3	74000	99
1900	63621	4.80360	t_{10}	$p_{10,11}$	1897	1900	1904	8	38.94486	4.86811	73810	71000	4.85126		80000	1900
01	69389	4.84129	r_{11}	$f_{10,11}$	1898	1901-02	1905	8	39.29271	4.91159	81580	79000	4.89763		86000	01
02	88767	4.94825										87000	4.93952		93000	02
03	100461	5.00200		$t_{10,11}$	1900	1903	1906	7	34.92817	4.98974	97670	98000	4.99123		102000	03
04	117081	5.06849	p_{11}	$r_{11,12}$	1902	1904	1906	5	25.28328	5.05666	113900	110000	5.04139	r_3	110000	04
05	134718	5.12943										125000	5.09691		120000	05
06	126494	5.13511	f_{11} t_{11} r_{12}	$p_{11,12}$	1904	1906	1907	4	20.53741	5.13435	136300	130000	5.11394		130000	06
07	160095	5.20438		$f_{11,12}$ $t_{11,12}$ $r_{12,13}$	1906 1909 1907	1907 1907-08 1908	1908 1909 1909	3 4 3	15.59119 20.85405 15.71894	5.19706 5.21351 5.23965	157400 163500 173600	160000	5.20412		142000	07
08	178527	5.25170	p_{12}									175000	5.24304		155000	08
09	183171	5.26286	f_{12} t_{12} r_{13}	$p_{12,13}$	1908	1909	1910	3	15.83586	5.27862	189900	190000	5.27875	p_3	168000	09
1910	209557	5.32130		$f_{12,13}$ $t_{12,13}$ $r_{13,14}$	1909 1909 1910	1910 1910-11 1911	1911 1912 1913	3 4 4	15.92747 21.27565 21.40802	5.30916 5.31891 5.35200	203800 208400 224900	205000	5.31175		185000	1910
11	220449	5.34331	p_{13} f_{13}									220000	5.34242		205000	11
12	222935	5.34818	t_{13}									233000	5.36736		225000	12
13	248446	5.39523	r_{14}	$p_{13,14}$	1911	1913	1914	4	21.51122	5.37780	238700	248000	5.39445	f_4	245000	13
14	265763	5.42450		$f_{13,14}$	1912	1914	1917	6	32.62046	5.43674	273400	263000	5.41996		270000	14
1915	281104	5.44887	p_{14}	$t_{13,14}$	1912	1915	1918	7	38.14671	5.44953	281500	280000	5.44716		298000	1915

Table M, Part a (concluded) Crude Petroleum Output

1 Year	2a or 4a Crude Petroleum Output	2b or 4b	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle	8	9	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Petroleum Output thousands barrels	12a Smoothing Line B	12b	13 Phase Point (major cycle)	24 SL M (thousands barrels)	1 Year
	thousands barrels	logarithm			Begin	Middle	End					thousands barrels	logarithm			
1916	300767	5.47823		$r_{14,15}$	1913	1916	1919	7	38.37645	5.48235	303600	300000	5.47712		325000	1916
17	335316	5.52545	f_{14}									325000	5.51188	t4	350000	17
18	335928	5.52625	t_{14}	$p_{14,15}$	1916	1917-18	1920	5	27.75418	5.55084	355500	355000	5.55023		380000	18
19	378372	5.57792	r_{15}	$f_{14,15}$	1918	1919	1920	3	16.75050	5.58350	383300	390000	5.59106		420000	19
1920	442929	5.64633	p_{15}	$t_{14,15}$	1918	1919-20	1921	4	22.42661	5.60665	404300	425000	5.62839		450000	1920
21	472183	5.67411	f_{15} t_{15}	$r_{15,16}$	1919	1920-21	1922	4	22.64463	5.66116	458300	495000	5.69461	r4	490000	21
22	557531	5.74627	r_{16}	$p_{15,16}$ $f_{15,16}$	1920 1921	1921-22 1922	1923 1923	4 3	22.93146 17.28513	5.73286 5.76171	540600 577700	560000	5.74819		530000	22
23	732407	5.86475	p_{16}	$t_{15,16}$	1921	1923	1925	5	29.02174	5.80435	637300	640000	5.80618		580000	23
24	713940	5.85366	f_{16}	$r_{16,17}$	1922	1924	1926	5	29.23461	5.84692	702900	710000	5.85126		625000	24
25	763743	5.88295		$p_{16,17}$	1923	1925	1927	5	29.44313	5.88863	773800	760000	5.88081		670000	25
26	770874	5.88698	t_{16}	$f_{16,17}$	1924	1925-26	1927	4	23.57838	5.89459	784500	810000	5.90849		(720000)	26
27	901129	5.95479	r_{17} p_{17}	$t_{16,17}$	1926	1927	1928	3	17.79672	5.93224	855500	865000	5.93702	p4	(760000)	27
28	901474	5.95495	f_{17} t_{17}	$r_{17,18}$ $p_{17,18}$	1927 1927	1927-28 1928	1928 1929	2 3	11.90974 17.91291	5.95487 5.97097	901300 935300	930000	5.96848		(810000)	28
29	1007323	6.00317	r_{18} p_{18}	$f_{17,18}$	1928	1929	1929	2	11.95812	5.97906	952900	950000	5.97772		(850000)	29
1930	898011	5.95328	f_{18}	$t_{17,18}$	1928	1930	1932	5	29.73633	5.94727	885700	920000	5.96379	f_5	(900000)	1930
31	851081	5.92997		$r_{18,19}$ $p_{18,19}$	1929 1929	1930-31 1931	1932 1933	4 5	23.78138 29.73834	5.94535 5.94767	881800 886500	880000	5.94448		(960000)	31
32	785159	5.89496	t_{18}	$f_{18,19}$	1930	1932	1933	4	23.73517	5.93879	868500	850000	5.92942		(1010000)	32
33	905656	5.95696	r_{19} p_{19}	$t_{18,19}$	1932	1933	1934	3	17.81004	5.93668	864300	870000	5.93952		(1070000)	33
34	908065	5.95812	f_{19} t_{19}	$r_{19,20}$ $p_{19,20}$	1933 1933	1933-34 1934	1934 1935	2 3	11.91508 17.91360	5.95754 5.97120	906900 935800	910000	5.95904	t_5	(1130000)	34
35	996596	5.99852	r_{20} p_{20}	$f_{19,20}$ $t_{19,20}$	1934 1934	1934-35 1935	1935 1936	2 3	11.95664 17.98790	5.97832 5.99597	951300 990800	980000	5.99123		(1180000)	35
36	1099687	6.03126	f_{20} t_{20}	$r_{20,21}$ $p_{20,21}$	1935 1935	1935-36 1936	1936 1937	2 3	12.02978 18.13670	6.01489 6.04557	1035000 1111000	1100000	6.04139		(1240000)	36
37	1279160	6.10692	r_{21} p_{21}	$f_{20,21}$	1936	1936-37	1937	2	12.13818	6.06909	1172000	(1190000)	6.07555		(1280000)	37
1938	1214355	6.08435	f_{21}									(1290000)	6.11059		(1350000)	1938

Table M. CRUDE PETROLEUM OUTPUT.

Part b part of the calculations for the second stage of smoothing

Columns numbered as in Tables B and C.

14 Cycle	15	16 Yearly Figures Included in the Cycle	17	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (thousands barrels)
	Begin	Middle	End				
$f_{1,2}$	1865	1874	1883	19	75.23477	3.95972	9114
$t_{1,2}$	1872	1878	1885	14	58.80610	4.20044	15865
$r_{1,2}$	1879	1884	1888	10	44.11309	4.41131	25782
$p_{1,2}$	1882	1887	1892	11	49.61995	4.51090	32427
$f_{2,3}$	1884	1889	1894	11	50.13292	4.55754	36100
$t_{2,3}$	1886	1892	1899	14	65.21637	4.65831	45530
$r_{2,3}$	1889	1896	1904	16	76.56579	4.78536	61000
$p_{2,3}$	1892	1900	1908	17	83.37010	4.90412	80190
$f_{3,4}$	1895	1904	1913	19	95.90487	5.04762	11600
$t_{3,4}$	1899	1908	1917	19	98.74280	5.19699	157400
$r_{3,4}$	1904	1912	1920	17	91.01993	5.35411	226000
$p_{3,4}$	1909	1918	1927	19	106.14709	5.58669	386100
$f_{4,5}$	1913	1922	1930	18	102.75680	5.70871	511300
$t_{4,5}$	1917	1925-26	1934	18	104.79057	5.82170	663300

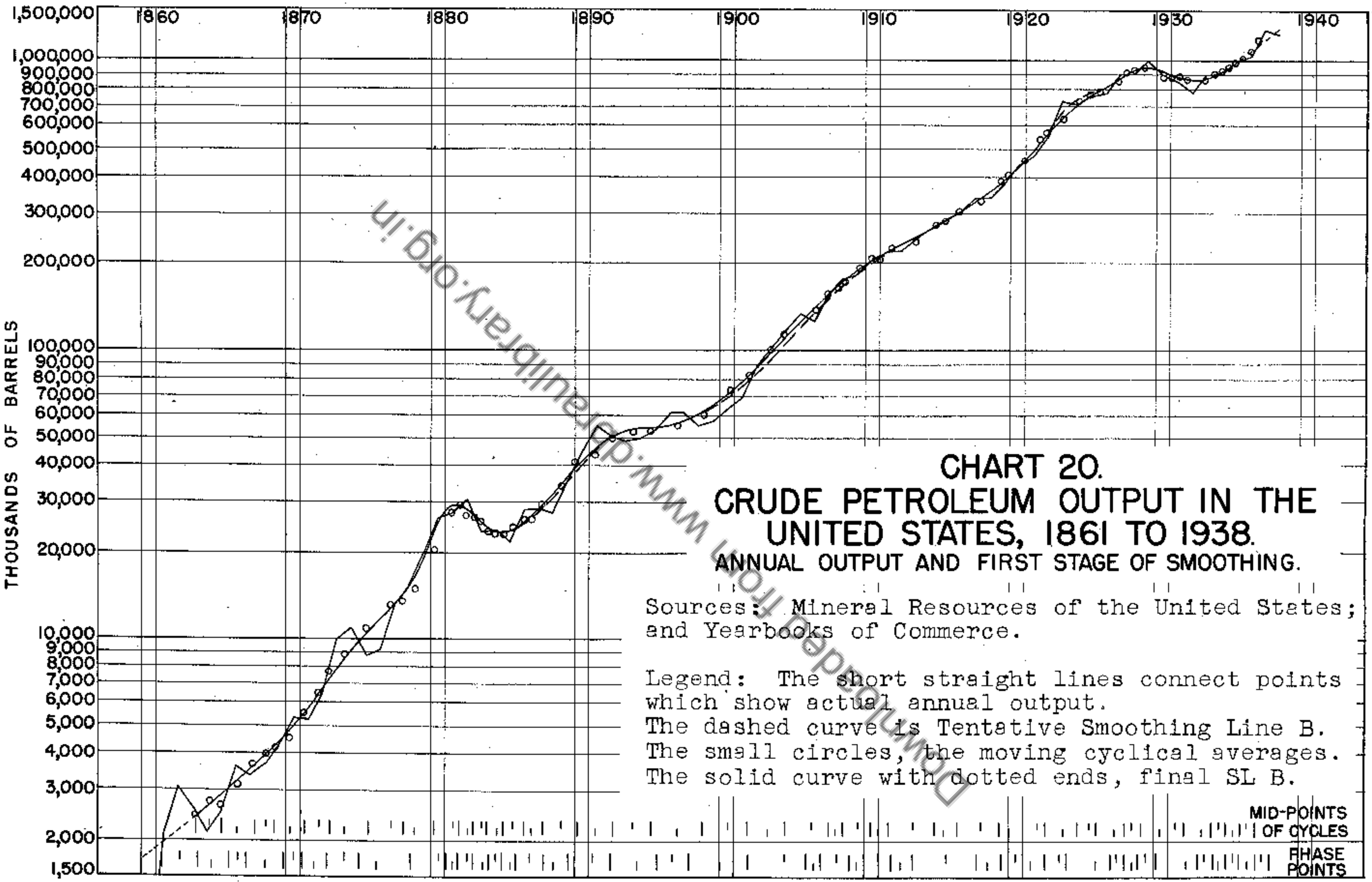


CHART 20.

CRUDE PETROLEUM OUTPUT IN THE UNITED STATES, 1861 TO 1938.

ANNUAL OUTPUT AND FIRST STAGE OF SMOOTHING.

Sources: Mineral Resources of the United States; and Yearbooks of Commerce.

Legend: The short straight lines connect points which show actual annual output.
The dashed curve is Tentative Smoothing Line B.
The small circles, the moving cyclical averages.
The solid curve with dotted ends, final SL B.

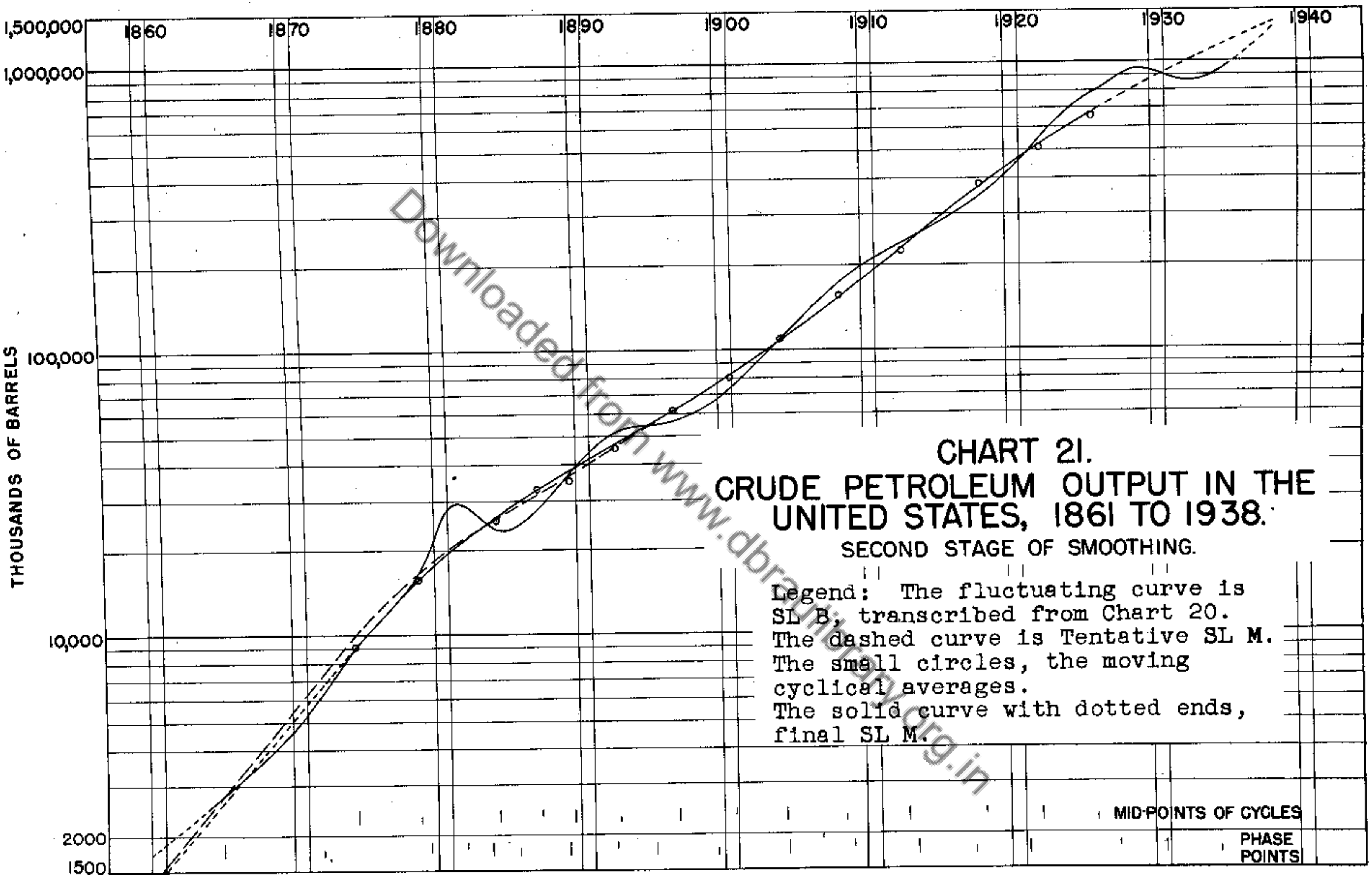


CHART 21.

CRUDE PETROLEUM OUTPUT IN THE UNITED STATES, 1861 TO 1938.

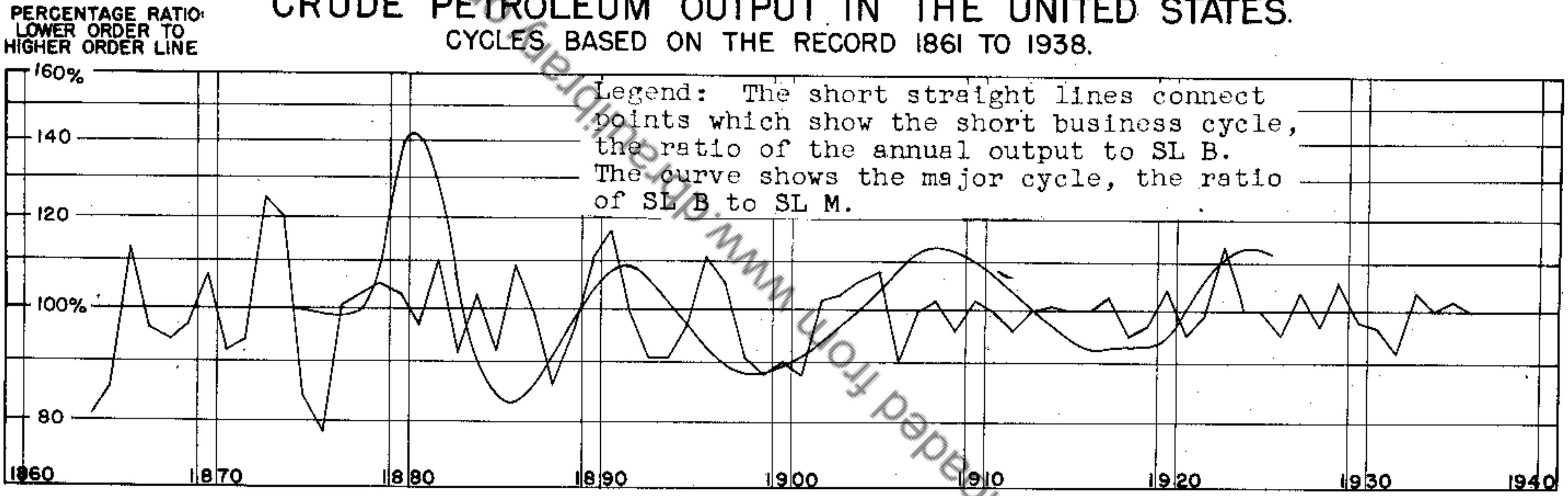
SECOND STAGE OF SMOOTHING.

Legend: The fluctuating curve is
SL B, transcribed from Chart 20.
The dashed curve is Tentative SL M.
The small circles, the moving
cyclical averages.
The solid curve with dotted ends,
final SL M.

CHART 22.

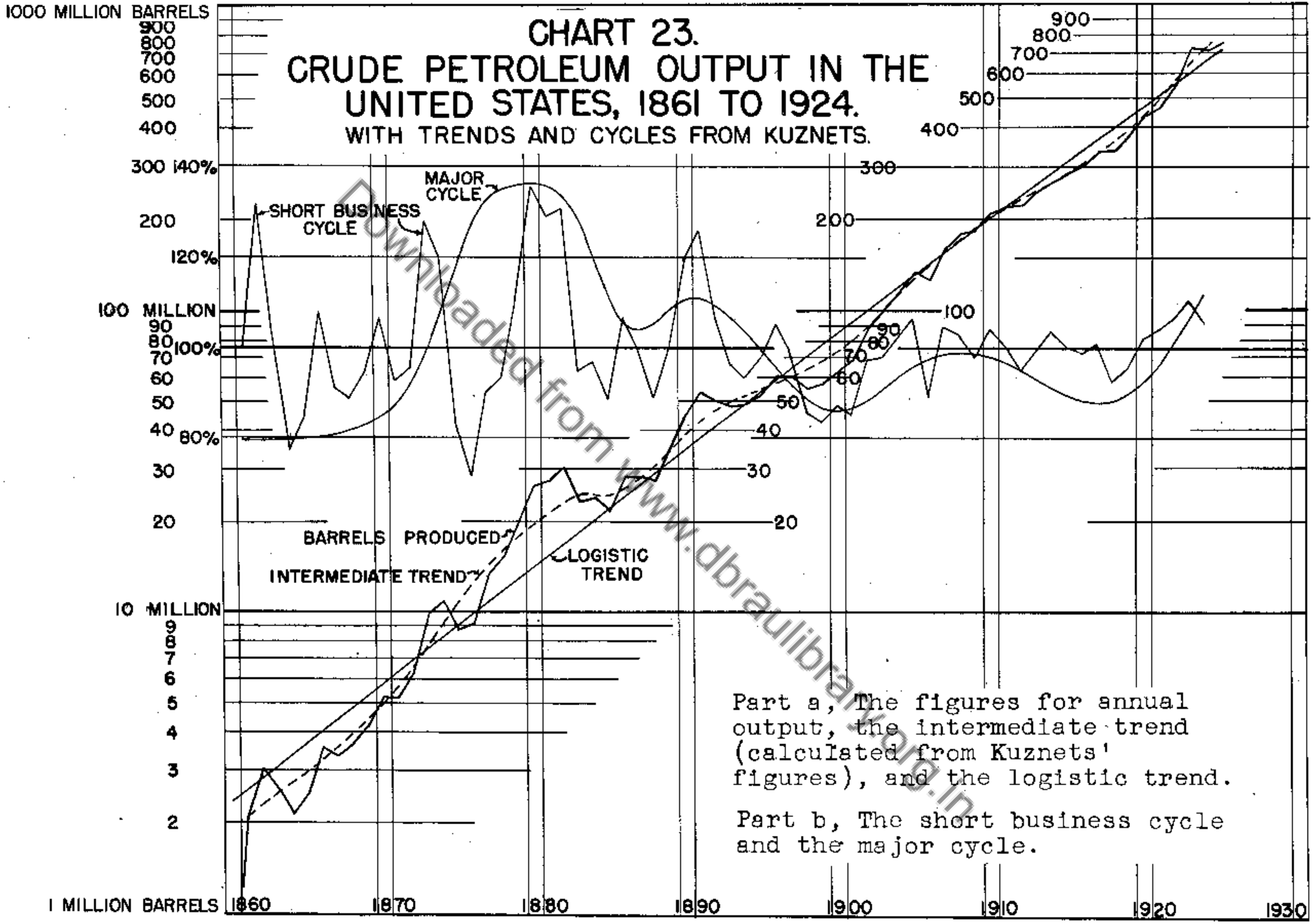
CRUDE PETROLEUM OUTPUT IN THE UNITED STATES.

CYCLES BASED ON THE RECORD 1861 TO 1938.



Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual output to SL B. The curve shows the major cycle, the ratio of SL B to SL M.

CHART 23.
CRUDE PETROLEUM OUTPUT IN THE UNITED STATES, 1861 TO 1924.
WITH TRENDS AND CYCLES FROM KUZNETS.



Part a, The figures for annual output, the intermediate trend (calculated from Kuznets' figures), and the logistic trend.

Part b, The short business cycle and the major cycle.

Table N. CRUDE PETROLEUM OUTPUT IN THE UNITED STATES

Calculation of standard measures of the two orders of cycles.

Based on the record 1861 to 1938. Three pages.

Columns numbered as in Tables D and E.

1 Year	2 (or 4)a Crude Petroleum Output	12 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of barrels)			The short business cycle			The major cycle			
1861	2114									1861
62	3057									62
63	2611									63
64	2116	2600		81	-19	361				64
65	2498	2880		87	-13	169				65
66	3598	3170		113	+13	169				66
67	3347	3500		96	- 4	16				67
68	3646	3900		94	- 6	36				68
69	4215	4350		97	- 3	9				69
1870	5261	4900		107	+ 7	49				1870
71	5205	5650		92	- 8	64				71
72	6293	6700		94	- 6	36				72
73	9894	7900		125	+25	625				73
74	10927	9100		120	+20	400				74
75	8788	10400	10400	84	-16	256	100	0	0	75
76	9133	11700	11800	78	-22	484	99	- 1	1	76
77	13350	13200	13300	101	+ 1	1	99	- 1	1	77
78	15397	15000	15200	103	+ 3	9	99	- 1	1	78
79	19914	19000	17000	105	+ 5	25	112	+12	144	79
1880	26286	25500	18600	103	+ 3	9	137	+37	1369	1880
81	27661	28500	20200	97	- 3	9	141	+41	1681	81
82	30350	28000	22000	110	+10	100	128	+28	784	82
83	23450	25500	24000	92	- 8	64	106	+ 6	36	83
84	24218	23500	26000	103	+ 3	9	90	-10	100	84
85	21859	23800	28000	92	- 8	64	85	-15	225	85
86	28065	25800	31000	109	+ 9	81	83	-17	289	86
87	28283	28500	33000	99	- 1	1	86	-14	196	87
88	27612	32000	35000	86	-14	196	91	- 9	81	88
89	35164	37000	37500	95	- 5	25	99	- 1	1	89
1890	45824	41500	40000	110	+10	100	104	+ 4	16	1890
91	54293	46500	43000	117	+17	289	108	+ 8	64	91
92	50515	50500	46000	100	0	0	110	+10	100	92
93	48431	53000	50000	91	- 9	81	106	+ 6	36	93
94	49345	54000	53000	91	- 9	81	102	+ 2	4	94
95	52892	54500	56000	97	- 3	9	97	- 3	9	95
1896	60960	55000	60000	111	+11	121	92	- 8	64	1896

Table N. (continued) Crude Petroleum Output in the United States

1 Year	2 (or 4)a Crude Petroleum Output	12 Smoothing Lino B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of barrels)			The short business cycle			The major cycle			
97	60476	57500	64000	105	+ 5	25	90	-10	100	97
98	55364	60500	69000	91	- 9	81	88	-12	144	98
99	57071	65000	74000	88	-12	144	88	-12	144	99
1900	63621	71000	80000	90	-10	100	89	-11	121	1900
01	69389	79000	86000	88	-12	144	92	- 8	64	01
02	88767	87000	93000	102	+ 2	4	94	- 6	36	02
03	100461	98000	102000	103	+ 3	9	96	- 4	16	03
04	117081	110000	110000	106	+ 6	36	100	0	0	04
05	134718	125000	120000	108	+ 8	64	104	+ 4	16	05
06	126494	140000	130000	90	-10	100	108	+ 8	64	06
07	160095	160000	142000	100	0	0	113	+13	169	07
08	178527	175000	155000	102	+ 2	4	113	+13	169	08
09	183171	190000	168000	96	- 4	16	113	+13	169	09
1910	209557	205000	185000	102	+ 2	4	111	+11	121	1910
11	220449	220000	205000	100	0	0	107	+ 7	49	11
12	222935	233000	225000	96	- 4	16	104	+ 4	16	12
13	248446	248000	245000	100	0	0	101	+ 1	1	13
14	265763	263000	270000	101	+ 1	1	97	- 3	9	14
15	281104	280000	298000	100	0	0	94	- 6	36 end of the short period for calculation of sd	15
16	300767	300000	325000	100	0	0	92	- 8	64	16
17	335316	325000	350000	103	+ 3	9	93	- 7	49	17
18	335928	355000	380000	95	- 5	25	93	- 7	49	18
19	378372	390000	420000	97	- 3	9	93	- 7	49	19
1920	442929	425000	450000	104	+ 4	16	94	- 6	36	1920
21	472183	495000	490000	95	- 5	25	101	+ 1	1	21
22	557531	560000	530000	99	- 1	1 end of the short period for calculation of sd	106	+ 6	36	22
23	732407	640000	580000	114	+14	196	110	+10	100	23
24	713940	710000	625000	100	0	0	114	+14	196	24
25	763743	760000	670000	100	0	0	113	+13	169	25
26	770874	810000		95	- 5	25				26
27	901129	865000		104	+ 4	16				27
28	901474	930000		97	- 3	9				28
29	1007323	950000		106	+ 6	36				29
1930	898011	920000		98	- 2	4				1930
31	851081	880000		97	- 3	9				31
32	785159	850000		92	- 8	64				32
33	905656	870000		104	+ 4	16				33
34	908065	910000		100	0	0				34
35	996596	980000		102	+ 2	4				35
36	1099687	1100000		100	0	0				36
37	1279160									37
1938	1214355									1938

$\Sigma(d^2) = 8679$

sd $= \sqrt{\frac{8679}{51}} = 12.90\%$

in the major cycle
(the full period)

$\Sigma(d^2) = 5160$

sd $= \sqrt{\frac{5160}{73}} = 8.41\%$, in the short business cycle.
(the full period)

(Table N is concluded on the next page)

Table N (concluded) Crude Petroleum Output

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles

of actual about SL B[28] (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1			.9	1.3
2	.6	.6	1.4	1.3
3	.7	.6	.9	.9
4	1.2	1.0	1.2	2.4
5	2.0	.5	.5	.4
6	.7	.4	.7	.4
7	.5	.5	.5	.4
8	.9	1.0	.7	1.1
9	1.7	1.3	1.9	1.5
10	1.0	1.2	2.2	2.0
11	2.5	1.5	.5	.5
12	1.2	.8	.6	.4
13	1.0	.9	.6	.8
14	2.4	2.1	.7	1.0
15	1.0	.5	.5	1.0
16	1.0	.5	2.0	1.0
17	.5	.5	.6	.4
18	.5	.7	2.0	.8
19	.5	.7	.3	.5
20	.5	.4	.6	.5
Average Length	1.1 yr.	.8 yr.	1.0 yr.	.9 yr.

Total length of typical short business cycle 3.8 yrs.

of SL B about SL M[33] (the major cycle)

1	2.5	1.8	2.0	3.5
2	3.4	2.7	4.3	5.0
3	4.5	4.5	4.0	3.8
4	6.3			
Average Length	4.2 yrs.	3.0 yrs.	3.4 yrs.	4.1 yrs.

Total length of typical major cycle 14.7 yrs.

Percentage Deviations at Peak and Trough

of actual from SL B[29] (the short business cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1864	-19%
1866	+13%	68	6
70	7	71	8
73	25	76	22
79	5	81	3
82	10	83	8
84	3	85	8
86	9	88	14
91	17	93	9
96	11	99	12
1905	8	1906	3
08	2	09	4
10	2	12	4
17	3	18	5
20	4	21	5
23	14	26	5
27	4	28	3
29	6	32	8
33	4	34	0
1935	2	1936	0
Average Deviation in the short business cycle	12.4% at peak		8.8% at trough

of SL B from SL M[34] (the major cycle)

1881	+42%	1886	-17%
1892	10	1899	12
1908	13	1916	8
Average deviation in the major cycle	21.7% at peak		12.3% at trough

Section 5. PIG IRON PRODUCTION.

COMPARISON of Kuznets' trend with the smoothing line SL M (thousands of long tons)

	Kuznets' Trend	SL M
1855	517	(470)
1860	769	(715)
1865	1140	(1080)
1870	1684	1660
1875	2475	2420
1880	3610	3430
1885	5209	4850
1890	7404	6900
1895	10312	9800
1900	13991	13600
1905	18381	18300
1910	23268	23600
1915	28306	29600
1920	33104	34600

Kuznets' equation:

$$y = \frac{50403}{1+10^{(1.80987-0.17431x)}}$$

x in units of 5 years; origin at 1860.

THE agreement in the trend lines obtained by the two methods is fairly close. SL M, from 1880 to 1900, falls below the value of the trend line. The major cycles obtained by the two methods are similar.

THE amplitude calculated for the method of successive smoothings, based on the period running to 1939, takes in the violent displacements of the recent years, and is consequently quite large.

Table P. PIG IRON PRODUCTION IN THE UNITED STATES, 1854 TO 1939

Two stages of smoothing.

Sources: Mineral Resources of the United States; Yearbooks of Commerce.

Part a (six pages)

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a
Year	Output of Pig Iron		Phase Point (short cycle)	Cycle	Yearly Figures Included in the Cycle			Length in Years	Moving Total of Logs
	thousands of long tons	logarithm			Begin	Middle	End		
1854	657								
55	700	2.84510	r_1						
56	789	2.89708	p_1						
57	713	2.85309	f_1	$r_{1,2}$	1855	1857	1858	4	11.39461
58	630	2.79934	t_1	$p_{1,2}$	1856	1858	1860	5	14.33949
59	751	2.87564	r_2	$f_{1,2}$	1857	1859	1860	4	11.44241
1860	821	2.91434	p_2 f_2	$t_{1,2}$	1858	1860	1861	4	11.40423
61	653	2.81491	t_2	$r_{2,3}$	1859	1861	1863	5	14.37922
62	703	2.84696		$p_{2,3}$ $f_{2,3}$	1860 1861	1862 1862-63	1864 1864	5 4	14.50962 11.59528
63	846	2.92737	r_3	$t_{2,3}$	1862	1863	1865	4	11.70049
64	1014	3.00604	p_3 f_3	$r_{3,4}$	1863	1864	1865	3	8.85353
65	832	2.92012	t_3 r_4	$p_{3,4}$	1864	1865	1866	3	9.00751
66	1206	3.08135	p_4	$f_{3,4}$ $t_{3,4}$	1865 1865	1866 1866-67	1867 1868	3 4	9.11708 12.27272
67	1305	3.11561	f_4	$r_{4,5}$	1866	1867	1868	3	9.35260
68	1431	3.15564	t_4 r_5	$p_{4,5}$ $f_{4,5}$	1867 1868	1868 1868-69	1869 1869	3 2	9.50450 6.38889
69	1711	3.23325	p_5 f_5	$t_{4,5}$	1869	1869-70	1870	2	6.45466
1870	1665	3.22141		$r_{5,6}$	1869	1870	1871	3	9.68689
71	1707	3.23223	t_5 r_6	$p_{5,6}$	1869	1871	1872	4	13.09326
72	2549	3.40637	p_6	$f_{5,6}$	1870	1872	1874	5	16.64881
73	2561	3.40841							
74	2401	3.38039	f_6	$t_{5,6}$	1871	1874	1876	6	20.00522
75	2024	3.30621		$r_{6,7}$	1872	1875	1879	8	26.88932
76	1869	3.27161	t_6						
77	2067	3.31534		$p_{6,7}$	1873	1877	1882	10	34.33806
78	2301	3.36192							
79	2742	3.43807	r_7	$f_{6,7}$	1875	1879	1883	9	31.22164
1880	3835	3.58377							

PIG IRON PRODUCTION

The values of the standard deviation, by the several calculations:

	From Kuznets' figures, based on period 1854 to 1924	Figures secured by the method of smoothing by stages	
		based on period 1854 to 1924	based on period 1854 to 1939
The short business cycle	14.8%	13.15%	16.6%
Years included (omit terminal half-cycle)	full	1857 to 1922	1857 to 1937
The major cycle	10.0%	9.0%	11.0%
Years included (omit terminal half-cycle)	full	1867 to 1919	1867 to 1929

Table P. PIG IRON PRODUCTION IN THE UNITED STATES, 1854 TO 1939

Two stages of smoothing.

Sources: Mineral Resources of the United States; Yearbooks of Commerce.

Part a (six pages)

1 Year	2a or 4a Output of Pig Iron thousands of long tons	2b or 4b Output of Pig Iron logarithm	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Yearly Figures Included in the Cycle Middle	9 Yearly Figures Included in the Cycle End	10 Length in Years	11a Moving Total of Logs
1854	657								
55	700	2.84510	r_1						
56	789	2.89708	p_1						
57	713	2.85309	f_1	$r_{1,2}$	1855	1857	1858	4	11.39461
58	630	2.79934	t_1	$p_{1,2}$	1856	1858	1860	5	14.33949
59	751	2.87564	r_2	$f_{1,2}$	1857	1859	1860	4	11.44241
1860	821	2.91434	p_2	$t_{1,2}$	1858	1860	1861	4	11.40423
61	653	2.81491	f_2 t_2	$r_{2,3}$	1859	1861	1863	5	14.37922
62	703	2.84696		$p_{2,3}$ $f_{2,3}$	1860 1861	1862 1862-63	1864 1864	5 4	14.50962 11.59528
63	846	2.92737	r_3	$t_{2,3}$	1862	1863	1865	4	11.70049
64	1014	3.00604	p_3	$r_{3,4}$	1863	1864	1865	3	8.85353
65	832	2.92012	f_3 t_3	$p_{3,4}$	1864	1865	1866	3	9.00751
66	1206	3.08135	r_4 p_4	$f_{3,4}$ $t_{3,4}$	1865 1865	1866 1866-67	1867 1868	3 4	9.11708 12.27272
67	1305	3.11561	f_4	$r_{4,5}$	1866	1867	1868	3	9.35260
68	1431	3.15564	t_4 r_5	$p_{4,5}$ $f_{4,5}$	1867 1868	1868 1868-69	1869 1869	3 2	9.50450 6.38889
69	1711	3.23325	p_5 f_5	$t_{4,5}$	1869	1869-70	1870	2	6.45466
1870	1665	3.22141		$r_{5,6}$	1869	1870	1871	3	9.68689
71	1707	3.23223	t_5 r_6	$p_{5,6}$	1869	1871	1872	4	13.09326
72	2549	3.40637	p_6	$f_{5,6}$	1870	1872	1874	5	16.64881
73	2561	3.40841							
74	2401	3.38039	f_6	$t_{5,6}$	1871	1874	1876	6	20.00522
75	2024	3.30621		$r_{6,7}$	1872	1875	1879	8	26.88932
76	1869	3.27161	t_6						
77	2067	3.31534		$p_{6,7}$	1873	1877	1882	10	34.33806
78	2301	3.36192							
79	2742	3.43807	r_7	$f_{6,7}$	1875	1879	1883	9	31.22164
1880	3835	3.58377							

Table P, Part a (continued) Pig Iron Production, two stages of smoothing

11b	11c	12a	12b	13	20a	20b	24	1
Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Pig Iron Output (1000's) long tons)	Smoothing Line B		Phase Point (major cycle)	Second Approximation to SL M		Final SL M (1000's) long tons)	Year
		thousands long tons	logarithm		thousands long tons	logarithm		
		(700)					(435)	1854
		(700)					etc.	55
		(700)						56
2.84865	706	700						57
2.86790	738	700						58
2.86060	725	705	2.84819	f_1				59
2.85106	710	720	2.85733		(not needed in these years)			1860
2.87584	751	740	2.86923				(column 20a accepted, until 1892)	61
2.90192 2.89882	798 792	770	2.88649					62
2.92512	842	830	2.91908					63
2.95118	894	880	2.94448	t_1				64
3.00250	1006	950	2.97772					65
3.03903 3.06818	1094 1170	1080	3.03342					66
3.11753	1311	1300	3.11394	r_1				67
3.16817 3.19445	1473 1565	1500	3.17609					68
		1650	3.21748					69
3.22733 3.22896	1688 1694	1800	3.25527	p_1				1870
3.27332	1876	1900	3.27875					71
3.32976	2137	2040	3.30963					72
		2120	3.32634					73
3.33420	2159	2220	3.34635					74
3.36118	2297	2350	3.37107	f_2				75
		2450	3.38917					76
3.43381	2715	2600	3.41497					77
		2800	3.44716					78
3.46907	2945	3000	3.47712	t_2			etc.	79
		3270	3.51455				3430	1880

(Table P, part a, is continued on next page)

Table P, Part a (continued) Pig Iron Production, two stages of smoothing

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a
Year	Output of Pig Iron		Phase Point	Cycle	Yearly Figures Included in the cycle			Length in Years	Moving Total of Logs
	thousands of long tons	logarithm			Begin	Middle	End		
1881	4144	3.61742		$t_{6,7}$	1877	1881	1884	8	28.25639
82	4623	3.66492	p_7						
				$r_{7,8}$	1880	1992-83	1885	6	21.74798
83	4596	3.66238							
			f_7						
84	4098	3.61257		$p_{7,8}$	1882	1884	1886	5	18.30137
85	4045	3.60692	t_7						
				$f_{7,8}$	1884	1885-86	1887	4	14.78140
86	5683	3.75458	r_8						
				$t_{7,8}$	1885	1886-87	1888	4	14.98108
87	6417	3.80733	p_8						
			f_8	$r_{8,9}$	1886	1887-88	1889	4	15.25520
88	6490	3.81225	t_8						
				$p_{8,9}$	1887	1888-89	1890	4	15.46455
89	7604	3.88104	r_9	$f_{8,9}$	1888	1889	1890	3	11.65722
1890	9203	3.96393	p_9	$t_{8,9}$	1889	1890	1891	3	11.76300
			f_9	$r_{9,10}$	1890	1890-91	1891	2	7.88196
91	8280	3.91803	t_9	$p_{9,10}$	1890	1891	1892	3	11.84371
			r_{10}	$f_{9,10}$	1891	1891-92	1892	2	7.87978
92	9157	3.96175	p_{10}						
			f_{10}	$t_{9,10}$	1892	1892-93	1893	2	7.81454
93	7125	3.95279		$r_{10,11}$	1892	1893	1894	3	11.63782
				$p_{10,11}$	1892	1893-94	1895	4	15.61307
94	6657	3.82328	t_{10}	$f_{10,11}$	1893	1894	1895	3	11.65132
			r_{11}						
95	9446	3.97525	p_{11}	$t_{10,11}$	1894	1895	1896	3	11.73419
			f_{11}						
96	8623	3.93566		$r_{11,12}$	1895	1896	1897	3	11.89557
			t_{11}						
97	9653	3.98466		$p_{11,12}$	1895	1897	1899	5	20.10070
				$f_{11,12}$	1896	1897-98	1899	4	16.12545
98	11774	4.07092	r_{12}						
				$t_{11,12}$	1897	1898-99	1900	4	16.32932
99	13621	4.13421	p_{12}						
			f_{12}	$r_{12,13}$	1898	1899-00	1901	4	16.54496
1900	13789	4.13953	t_{12}						
				$p_{12,13}$	1899	1900-01	1902	4	16.72497
01	15878	4.20030	r_{13}	$f_{12,13}$	1900	1901-02	1903	4	16.84625
02	17821	4.25093	p_{13}	$t_{12,13}$	1901	1902	1903	3	12.70672
				$r_{13,14}$	1901	1902-03	1904	4	16.92365
03	18009	4.25549	f_{13}						
04	16479	4.21693	t_{13}						
			r_{14}	$p_{13,14}$	1903	1904-05	1906	4	17.19182
05	22992	4.31616		$f_{13,14}$	1904	1905	1907	4	17.34765
06	25307	4.40324	p_{14}	$t_{13,14}$	1904	1906	1908	5	21.55003
				$r_{14,15}$	1905	1906-07	1908	4	17.33310
07	25781	4.41132	f_{14}						
08	15936	4.20238	t_{14}	$p_{14,15}$	1907	1908	1909	3	13.02524
1909	25795	4.41154	r_{15}	$f_{14,15}$	1908	1909	1910	3	13.05015

Table P, Part a (continued) Pig Iron Production, two stages of smoothing

11b	11c	12a	12b	13	20a	20b	24	1
Moving Cyclical Average of Logs	Moving Cyclical Geometric Mean of Pig Iron Output (1000's long tons)	Smoothing Line B		Phase Point (major cycle)	Second Approximation to SL M		Final SL M (1000's long tons)	Year
		thousands long tons	logarithm		thousands long tons	logarithm		
3.53205	3404	3550	3.55023				3690	1881
		3850	3.58546				3950	82
3.62466	4214							
		4200	3.62325				4250	83
3.66027	4574	4500	3.65321				4500	84
		4900	3.69020				4850	85
3.69535	4958							
		5400	3.73239				5200	86
3.74527	5562							
		6000	3.77815				5600	87
3.81380	6513							
		6900	3.83885				6000	88
3.86614	7348							
3.88574	7687	7700	3.88649				6450	89
3.92100	8337	8400	3.92428	P_2			6900	1890
3.94098	8729							
3.94790	8870	8800	3.94448				7400	91
3.93989	8707							
		8200	3.91381	f_3	7900	3.89763	7900	92
3.90727	8077							
3.87927	7573	7500	3.87506		8450	3.92686	8500	93
3.90436	8023							
3.88377	7652	7500	3.87506	t_3	9000	3.95424	9100	94
3.91140	8155	8200	3.91381		9700	3.98677	9800	95
3.96519	9230	9200	3.96379		10250	4.00072	10500	96
4.02014	10475	10200	4.00860		11000	4.04139	11200	97
4.03136	10749			r_3				
		11400	4.05690		11700	4.06819	12000	98
4.08233	12087							
		12900	4.11059		12500	4.09691	12800	99
4.13624	13685							
		14400	4.15836		13300	4.12385	13600	1900
4.18124	15179							
		15700	4.19590		14100	4.14922	14400	01
4.21156	16277							
4.23557	17202	16900	4.22789		15000	4.17609	15300	02
4.23091	17018							
		18200	4.26007		15800	4.19866	16200	03
		19400	4.28780		16700	4.22272	17200	04
4.29796	19860							
4.33691	21722	20500	4.31175		17500	4.24304	18300	05
4.31001	20420	21100	4.32428		18500	4.26717	19300	06
4.33328	21541							
		21700	4.33646		19400	4.28780	20400	07
4.34175	21966	22100	4.34439		20300	4.30750	21500	08
4.35005	22390	23200	4.36549	p_3	21200	4.32634	22500	1909

(Table P, part a, in continued on next page.)

Table P, Part a (continued) Pig Iron Production, two stages of smoothing

1	2a or 4a	2b or 4b	5	6	7	8	9	10	11a
Year	Output of Pig Iron		Phase Point	Cycle	Yearly Figures Included in the Cycle			Length in Years	Moving Total of Logs
	thousands of long tons	logarithm			Begin	Middle	End		
				$t_{14,15}$	1908	1909-10	1911	4	17.42398
1910	27304	4.43623	p_{15}	$r_{15,16}$	1909	1910	1911	3	13.22160
			f_{15}						
11	23650	4.37383	t_{15}	$p_{15,16}$	1910	1911	1912	3	13.28321
12	29727	4.47315	r_{16}	$f_{15,16}$	1911	1912	1913	3	13.33786
				$t_{15,16}$	1911	1912-13	1914	4	17.70581
13	30966	4.49088	p_{16}						
			f_{16}	$r_{16,17}$	1912	1913-14	1915	4	17.80788
14	23332	4.36795	t_{16}						
15	29916	4.47590	r_{17}	$p_{16,17}$	1913	1915	1916	4	17.93061
				$f_{16,17}$	1914	1916	1918	5	22.61823
16	39435	4.59588							
				$t_{16,17}$	1914	1916-17	1919	6	27.10980
17	38621	4.58682	p_{17}						
				$r_{17,18}$	1916	1917-18	1919	4	18.26595
18	39055	4.59168							
				$p_{17,18}$	1917	1918-19	1920	4	18.23740
			f_{17}						
19	31015	4149157	t_{17}						
				$f_{17,18}$	1919	1919-20	1920	2	9.05890
			r_{18}						
				$t_{17,18}$	1919	1920	1921	3	13.28130
1920	36926	4.56733	p_{18}						
			f_{18}						
				$r_{18,19}$	1920	1921	1922	3	13.22462
21	16688	4.22240	t_{18}						
				$p_{18,19}$	1920	1921-22	1923	4	17.83058
				$f_{18,19}$	1921	1922	1923	3	13.26325
22	27220	4.43489	r_{19}						
				$t_{18,19}$	1921	1922-23	1924	4	17.82793
23	40361	4.60596	p_{19}	$r_{19,20}$	1922	1923-24	1925	4	18.10254
24	31406	4.49701	f_{19}	$p_{19,20}$	1923	1924-25	1926	4	18.25949
			t_{19}	$f_{19,20}$	1924	1925	1926	3	13.65353
25	36701	4.56468	r_{20}	$t_{19,20}$	1924	1925-26	1927	4	18.21661
26	39070	4.59184	p_{20}	$r_{20,21}$	1925	1926-27	1928	4	18.30116
			f_{20}						
27	36566	4.56308	t_{20}						
				$p_{20,21}$	1926	1927-28	1929	4	18.36603
28	38156	4.58156	r_{21}						
				$f_{20,21}$	1927	1928-29	1930	4	18.27596
29	42614	4.62955	p_{21}	$t_{20,21}$	1927	1929-30	1932	6	26.48493
1930	31752	4.50177							
			f_{21}						
31	18426	4.26543		$r_{21,22}$	1928	1931-32	1935	8	34.58495
32	8781	3.94354	t_{21}						
				$p_{21,22}$	1929	1933	1936	8	34.49516
33	13346	4.12535		$f_{21,22}$	1931	1933-34	1937	7	29.93353
34	16139	4.20788							
				$t_{21,22}$	1932	1935	1938	7	29.95052
35	21373	4.32987	r_{22}						
36	31029	4.49177							
				$r_{22,23}$	1935	1937	1938	4	17.67375
37	37127	4.56969	p_{22}						
			f_{22}						
38	19161	4.28242	t_{22}						
			r_{23}						
1939	35317								

Table P, Part a (concluded) Pig Iron Production, two stages of smoothing

11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Pig Iron Output (1000's long tons)	12a Smoothing Line B thousands long tons	12b Smoothing Line B logarithm	13 Phase Point (major cycle)	20a Second Approximation to SL M thousands long tons	20b Second Approximation to SL M logarithm	24 Final SL M (1000's long tons)	1 Year
4.35599	22698							
4.40720	25539	24200	4.38382		22300	4.34830	23600	1910
4.42140	26388	25600	4.40824		23200	4.36549	24700	11
4.44595	27922	27000	4.43136		24200	4.38382	25900	12
4.42645	26696							
		28000	4.44716		25200	4.40140	27000	13
4.45197	28312							
		29700	4.47276		26200	4.41830	28200	14
4.48265	30384	31400	4.49693		27200	4.43457	29600	15
4.52365	33392	33000	4.51851		28200	4.45025	30600	16
4.51830	32984							
		35000	4.54407		29300	4.46687	31600	17
4.56644	36850							
		36000	4.55630		30400	4.48287	32500	18
4.55935	36253							
		34500	4.53782		31300	4.49554	33700	19
4.52945	33842							
4.42710	26736	28000	4.44716	f_4	32200	4.50786	34600	1920
4.40821	25598	26500	4.42325		33000	4.51851	35200	21
4.45865	28751							
4.42108	26368	27000	4.43136	t_4	33400	4.52375	35300	22
4.45698	28640							
		30400	4.48287	r_4	33600	4.52634	34900	23
4.52564	33546							
		34000	4.53148		33000	4.51851	34300	24
4.56487	36717							
4.55118	35578	36400	4.56120		32300	4.50920	33600	25
4.55415	35822							
		37800	4.57749		31400	4.49693	32800	26
4.57529	37609							
		38000	4.57978	p_4	30400	4.48287	31700	27
4.59151	39040							
		37800	4.57749		29000	4.46240	30200	28
4.57399	37496							
		34000	4.53148		27700	4.44248	28900	29
4.41416	25951							
		27000	4.43136	f_5	26500	4.42325	(27300)	1930
		21000	4.32222		25200	4.40140	(25700)	31
4.32312	21043							
		18000	4.25527		24000	4.38021	(24300)	32
4.31189	20506	17400	4.24055	t_5	22800	4.35794	(23000)	33
4.27622	18889							
		18000	4.25527		21800	4.33846	(21900)	34
4.27850	18989	19300	4.28556		21000	4.32222	(21000)	35
		21700	4.33646	r_5	20000	4.30103	(20000)	36
4.41844	26208	24000					(19200)	37
		(26500)					(18600)	38
		(28000)					(18000)	1939

Table P. PIG IRON PRODUCTION

Part b part of the calculations for the second stage of smoothing, including correction for curvature.

14 Cycle	15 Yearly Figures Included in the Cycle Begin	16 Middle	17 End	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (first application 1000's long tons)	20a (to Table Pa) Second Approximation to SL M	20b Log of Second Approximation	21a Moving Total of Logs of Second Approximation	21b mca of Logs of Second Approx.	22 Difference in Logs (Column 19b minus col.21b)	23a Adjusted Moving Cyclical Average of Logs	23b Adjusted Moving Cyclical Geometric Mean (1000's long tons)	24 (to Table Pa) Final SL M	16 Year
$f_{1,2}$	1859	1867	1874	16	49.35797	3.08499	1216							1216		1867
$t_{1,2}$	1864	1871	1878	15	48.60184	3.24012	1738							1738		1871
$r_{1,2}$	1868	1875	1883	16	54.27289	3.39206	2466							2466		1875
$p_{1,2}$	1870	1880	1890	21	74.39289	3.54252	3488							3488		1880
$f_{2,3}$	1875	1883-84	1892	18	65.73484	3.65194	4487							4487		1883-84
$t_{2,3}$	1879	1886-87	1894	16	59.86259	3.74141	5513							5513		1886-87
$r_{2,3}$	1883	1890	1897	15	57.62143	3.84143	6941							6941		1890
$p_{2,3}$	1890	1899-00	1909	20	82.39877	4.11994	13181							13181		1899-00
$f_{3,4}$	1893	1906	1919	27	115.41317	4.27456	18817			114.62488	4.24537	.02919	4.30375	20126		1906
$t_{3,4}$	1895	1908	1921	27	116.53346	4.31605	20704			115.77015	4.28778	.02827	4.34432	22096		1908
$r_{3,4}$	1898	1911	1923	26	113.56149	4.36775	23321			112.79136	4.33813	.02962	4.39737	24967		1911
$p_{3,4}$	1909	1918	1927	19	85.19695	4.48405	30482			84.65772	4.45567	.02838	4.51243	32541		1918
$f_{4,5}$	1920	1925	1929	10	45.14346	4.51435	32685			44.98885	4.49889	.01546	4.52981	33870		1925
$t_{4,5}$	1922	1927	1932	11	49.28190	4.48017	30211			49.16734	4.46976	.01041	4.49058	30944		1927
$r_{4,5}$	1923	1929-30	1936	14	61.96838	4.42631	26688			61.96065	4.42576	.00055	4.42686	26722		1929-30

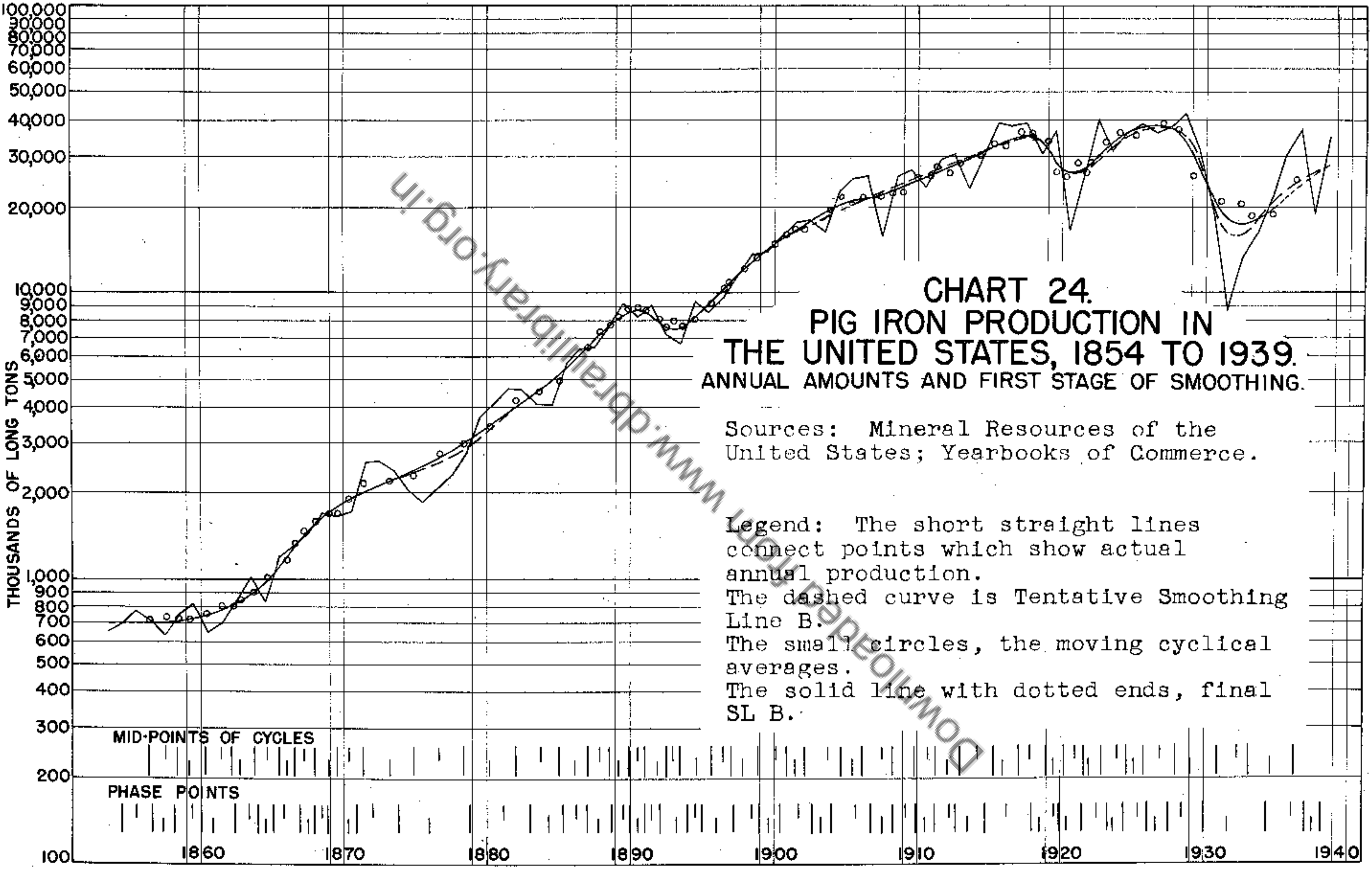


CHART 24.

PIG IRON PRODUCTION IN THE UNITED STATES, 1854 TO 1939.

ANNUAL AMOUNTS AND FIRST STAGE OF SMOOTHING.

Sources: Mineral Resources of the United States; Yearbooks of Commerce.

Legend: The short straight lines connect points which show actual annual production.
The dashed curve is Tentative Smoothing Line B.
The small circles, the moving cyclical averages.
The solid line with dotted ends, final SL B.

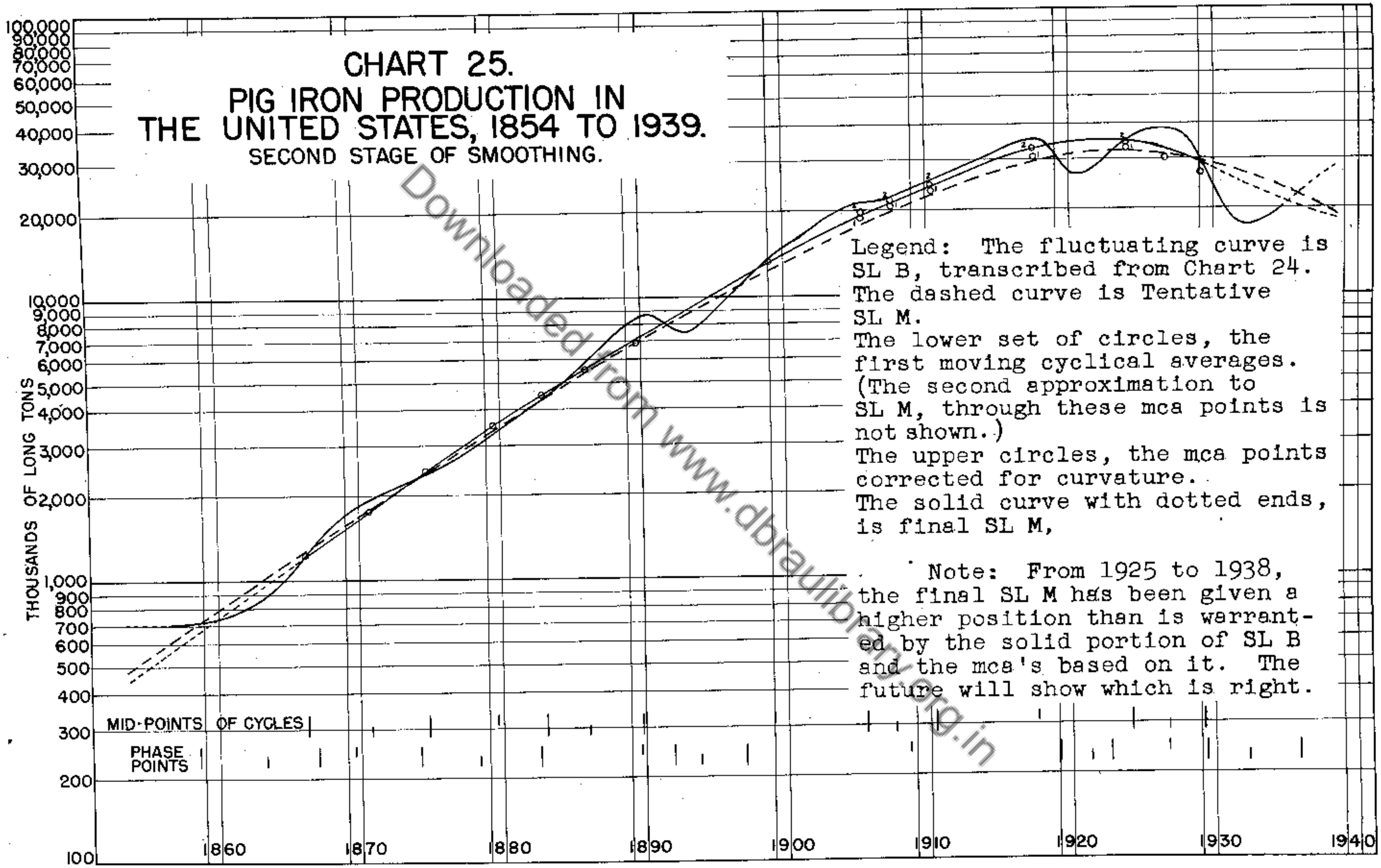


CHART 25.

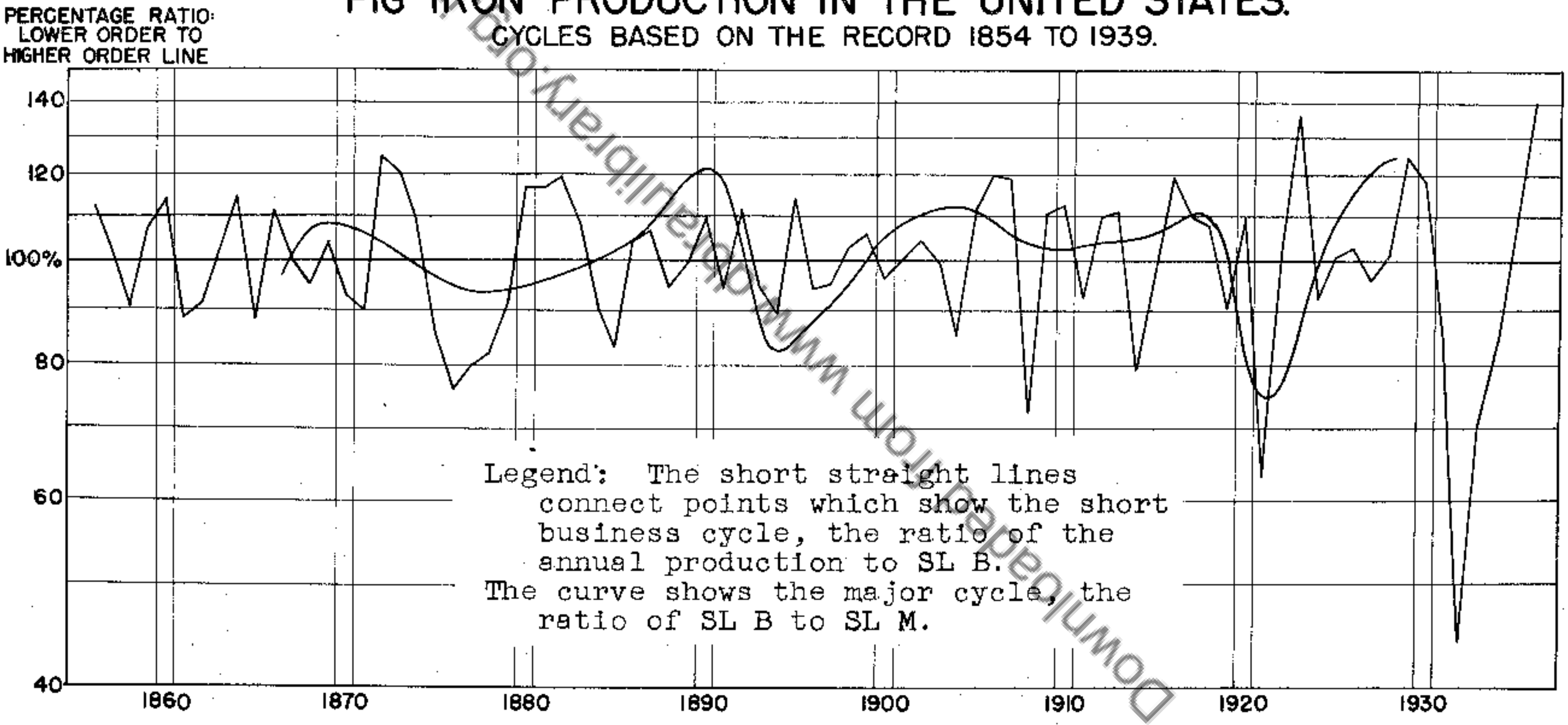
PIG IRON PRODUCTION IN THE UNITED STATES, 1854 TO 1939.

SECOND STAGE OF SMOOTHING.

Legend: The fluctuating curve is SL B, transcribed from Chart 24.
The dashed curve is Tentative SL M.
The lower set of circles, the first moving cyclical averages. (The second approximation to SL M, through these mca points is not shown.)
The upper circles, the mca points corrected for curvature.
The solid curve with dotted ends, is final SL M,

Note: From 1925 to 1938, the final SL M has been given a higher position than is warranted by the solid portion of SL B and the mca's based on it. The future will show which is right.

CHART 26.
PIG IRON PRODUCTION IN THE UNITED STATES.
CYCLES BASED ON THE RECORD 1854 TO 1939.



Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual production to SL B.
The curve shows the major cycle, the ratio of SL B to SL M.

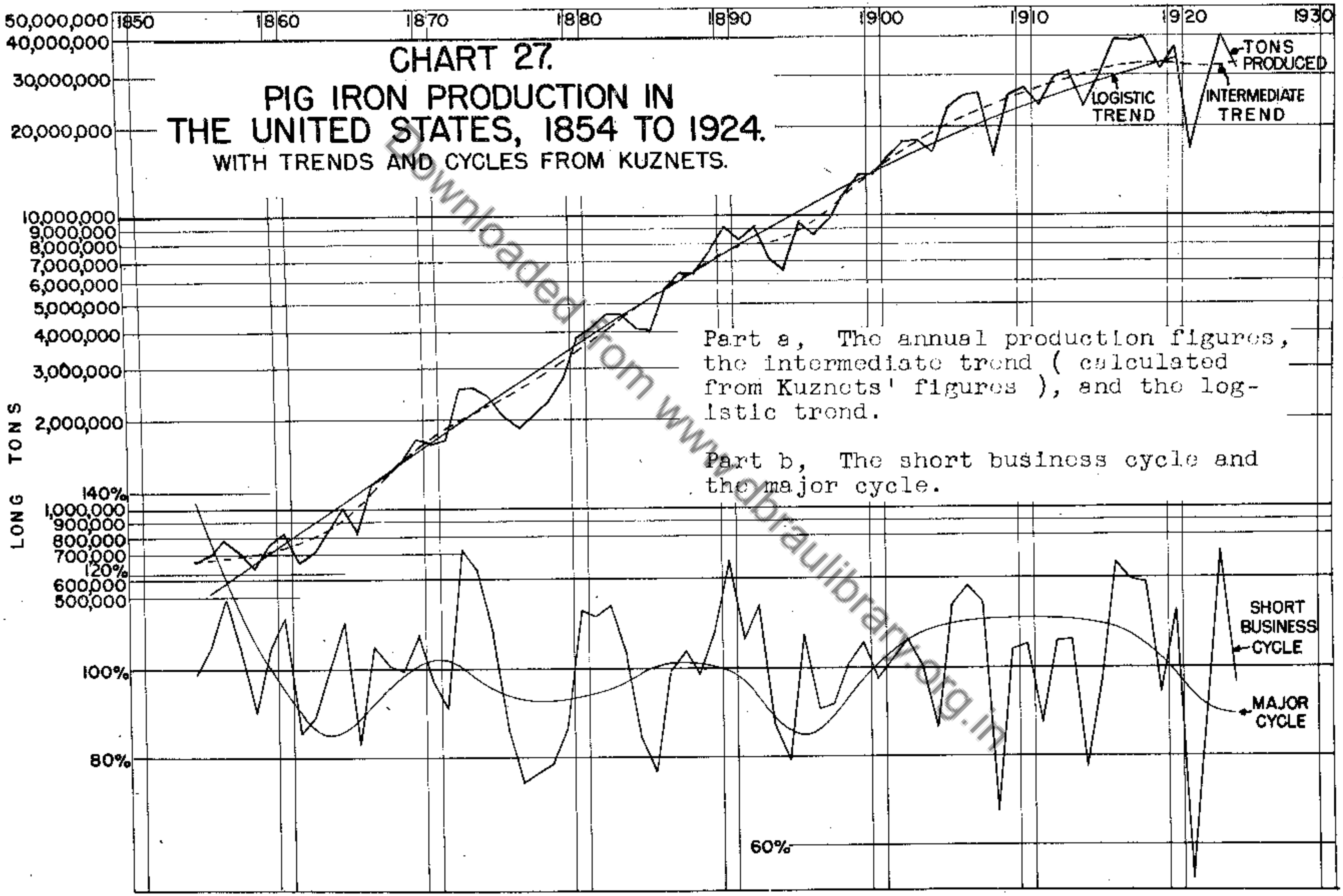


Table Q. PIG IRON PRODUCTION IN THE UNITED STATES

Calculation of standard measures of the two orders of cycles.

Based on the record 1854 to 1939. (Three pages.)

1 Year	2a or 4a Output of Pig Iron	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of long tons)			The short business cycle.			The major cycle			
1854	657									1854
55	700									55
56	789									56
57	713	700		102	+ 2	4				57
58	630	700		90	-10	100				58
59	751	705		107	+ 7	49				59
1860	821	720		114	+14	196				1860
61	653	740		88	-12	144				61
62	703	770		91	- 9	81				62
63	846	830		102	+ 2	4				63
64	1014	880		115	+15	225				64
65	832	950		88	-12	144				65
66	1206	1080		112	+12	144				66
67	1305	1300	1280	100	0	0	102	+ 2	4	67
68	1431	1500	1400	95	- 5	25	107	+ 7	49	68
69	1711	1650	1520	104	+ 4	16	109	+ 9	81	69
1870	1665	1800	1660	93	- 7	49	108	+ 8	64	1870
71	1707	1900	1800	90	-10	100	106	+ 6	36	71
72	2549	2040	1940	125	+25	625	105	+ 5	25	72
73	2561	2120	2080	121	+21	441	102	+ 2	4	73
74	2401	2220	2260	108	+ 8	64	98	- 2	4	74
75	2024	2350	2420	86	-14	196	97	- 3	9	75
76	1869	2450	2600	76	-24	576	94	- 6	36	76
77	2067	2600	2780	80	-20	400	93	- 9	81	77
78	2301	2800	3000	82	-18	324	93	- 9	81	78
79	2742	3000	3200	91	- 9	81	94	- 6	36	79
1880	3835	3270	3430	117	+17	289	95	- 5	25	1880
81	4144	3550	3690	117	+17	289	96	- 4	16	81
82	4623	3850	3950	120	+20	400	97	- 3	9	82
83	4596	4200	4250	109	+ 9	81	99	- 1	1	83
84	4098	4500	4500	91	- 9	81	100	0	0	84
85	4045	4900	4850	83	-17	289	101	+ 1	1	85
86	5683	5400	5200	105	+ 5	25	104	+ 4	16	86
87	6417	6000	5600	107	+ 7	49	107	+ 7	49	87
88	6490	6900	6000	94	- 6	36	115	+15	225	88
89	7604	7700	6450	99	- 1	1	119	+19	361	89
1890	9203	8400	6900	110	+10	100	122	+22	484	1890
91	8280	8800	7400	94	- 6	36	119	+19	361	91
92	9157	8200	7900	112	+12	144	104	+ 4	16	92

Table Q (continued) Pig Iron Production in the United States

1 Year	2a or 4a Output of Pig Iron	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of long tons)									
1893	7125	7500	8500	95	- 5	25	88	-12	144	1893
94	6657	7500	9100	89	-11	121	82	-18	324	94
95	9446	8200	9800	115	+15	225	84	-16	256	95
96	8623	9200	10500	94	- 6	36	88	-12	144	96
97	9653	10200	11200	95	- 5	25	91	- 9	81	97
98	11774	11400	12000	103	+ 3	9	95	- 5	25	98
99	13621	12900	12800	106	+ 6	36	101	+ 1	1	99
1900	13789	14400	13600	96	- 4	16	106	+ 6	36	1900
01	15878	15700	14400	101	+ 1	1	109	+ 9	81	01
02	17821	16900	15300	105	+ 5	25	110	+10	100	02
03	18009	18200	16200	99	- 1	1	112	+12	144	03
04	16479	19400	17200	85	-15	225	113	+13	169	04
05	22992	20500	18300	112	+12	144	112	+12	144	05
06	25307	21100	19300	120	+20	400	109	+ 9	81	06
07	25781	21700	20400	119	+19	361	106	+ 6	36	07
08	15936	22100	21500	72	-28	784	103	+ 3	9	08
09	25795	23200	22500	111	+11	121	103	+ 3	9	09
1910	27304	24200	23600	113	+13	169	102	+ 2	4	1910
11	23650	25600	24700	92	- 8	64	104	+ 4	16	11
12	29727	27000	25900	110	+10	100	104	+ 4	16	12
13	30966	28000	27000	111	+11	121	104	+ 4	16	13
14	23332	29700	28200	79	-21	441	105	+ 5	25	14
15	29916	31400	29600	95	- 5	25	106	+ 6	36	15
16	39435	33000	30600	120	+20	400	108	+ 8	64	16
17	38621	35000	31600	110	+10	100	111	+11	121	17
18	39055	36000	32500	108	+ 8	64	111	+11	121	18
19	31015	34500	33700	90	-10	100	102	+ 2	4	19
1920	36926	28000	34600	110	+10	100	81	-19	361	1920
21	16688	26500	35200	63	-37	1369	75	-25	625	21
22	27220	27000	35300	101	+ 1	1	76	-24	576	22
23	40361	30400	34900	137	+37	1369	87	-13	169	23
24	31406	34000	34300	92	- 8	64	99	- 1	1	24
25	36701	36400	33600	101	+ 1	1	108	+ 8	64	25
26	39070	37800	32800	103	+ 3	9	115	+15	225	26
27	36566	38000	31700	96	- 4	16	120	+20	400	27
28	38156	37800	30200	101	+ 1	1	125	+25	625	28
29	42614	34000	28900	125	+25	625	118	+18	324	29
1930	31752	27000		118	+18	324				1930
31	18426	21000		88	-12	144				31
32	8781	18000		49	-51	2601				32
33	13346	17400		77	-23	529				33
34	16139	18000		90	-10	100				34
35	21373	19300		111	+11	121				35
36	31029	21700		143	+43	1849				36
37	37127	24000		155	+55	3025				37
38	19161									38
1939	35317									1939

end of the short period for calculation of sd

$\Sigma(d^2) = 22195$

end of the short period for calculation of sd.

$\Sigma(d^2) = 7611$

$sd = \sqrt{\frac{7611}{63}} = 11.0\%$, in the major cycle. (the full period)

(Table Q is concluded on the next page)

$sd = \sqrt{\frac{22195}{81}} = 16.6\%$, in the short business cycle. (the full period)

Table Q (concluded) Pig Iron Production

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles

of actual about SL B[28] (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1	-	-	.7	.8
2	1.2	.7	.8	1.5
3	1.0	.7	.5	.6
4	.8	1.1	.7	.4
5	.6	.5	1.4	.6
6	1.2	1.8	2.2	2.9
7	2.3	1.9	1.3	1.0
8	.9	.7	.8	1.0
9	.8	.5	.5	.5
10	.5	.6	1.2	.8
11	.5	.5	1.0	1.3
12	1.3	.5	.7	.8
13	1.3	1.0	.5	.7
14	1.7	1.0	.8	.8
15	.9	.7	.6	.8
16	1.0	.4	.9	1.2
17	1.5	1.8	.5	.5
18	.5	.4	.6	1.0
19	1.0	.8	.3	.7
20	1.0	.6	.6	.9
21	.9	1.5	1.7	2.7
22	1.7	.9	-	-
Average length	1.1 yr.	.9 yr.	.9 yr.	1.0 yr.

Total Length of typical short business cycle 3.9 yrs.

of SL B about SL M[33] (the major cycle)

1	2.7	4.6	4.1	4.3
2	7.0	2.3	2.3	3.2
3	11.6	10.5	2.2	1.5
4	4.0	2.7		
Average length	6.3 yrs.	5.0 yrs.	2.9 yrs.	3.0 yrs.

Total length of typical major cycle 17.2 yrs.

Percentage Deviations at Peaks and Troughs

of actual from SL B[29] (the short business cycle)

At Peak Year	Deviation	At Trough Year	Deviation
		1858	-10%
1860	+14%	61	12
64	15	65	12
66	12	68	5
69	4	71	10
72	25	76	24
82	20	85	17
87	7	88	6
90	10	91	6
92	12	94	11
95	15	96	6
99	6	1900	4
1902	5	04	15
06	20	08	28
10	13	11	8
13	11	14	21
16	20	19	10
20	10	21	37
23	37	24	8
26	3	27	4
1929	25	1932	51
Average deviation in the short business cycle	14.7% at peak		14.5% at trough

of SL B from SL M[34] (the major cycle)

1869	9%	1877-78	9%
1890	22	1894	18
1904	13	1921	25
1928	25		
Average deviation in the major cycle	17.2%		13.0%

Section 6. PORTLAND CEMENT PRODUCTION.

COMPARISON of Kuznets' trend line with the smoothing line SL M (thousands of barrels)

	Kuznets' Trend	SL M
1880	36.9	(35)
1882	62.4	(52)
1885	136.8	(103)
1887	231.0	(168)
1890	506.0	(365)
1892	852.8	(660)
1895	1859.2	1630
1897	3113.5	3000
1900	6667.6	7300
1902	10900.8	12000
1905	22041.7	22500
1907	33775.5	32800
1910	58294	51000
1912	77498	65500
1915	104776	89000
1917	119069	102000
1920	133461	118000
1922	139004	127000
1925	144107	133000

Kuznets' equation:

$$y = \frac{148,481}{1 + 10^{(3.60421 - 0.56912x)}}$$

x in units of 5 years; origin at 1880.

IN the period 1910 to 1924 there was sharp curvature on the semi-logarithmic chart. Following the usual correction for curvature, SL M was adjusted to take a position toward the convex side from the first set of moving cyclical average points, but even at that, the values of SL M are almost uniformly smaller than the values of Kuznets' mathematical trend line. Only for the years 1897 to 1907 is there approximate equality between them. After 1924 the great depression lying in the future begins to cause a drop in SL M.

THERE is close similarity in the appearance of the major cycle by the two methods.

IT will be noted that SL B for this series was drawn by inspection without the objective check of a moving average. Since the chart was so free from construction lines, it was possible to proceed on the same sheet to locate SL M. This permits the whole graphical process to be viewed together on the finished chart (as it usually is on worksheets).

Values of the standard deviation, by the several calculations:

	From Kuznets' figures, based on period 1880 to 1924	Figures secured by the method of smoothing by stages	
		based on period 1880 to 1925	based on period 1880 to 1938
The short business cycle	8.7%	6.5%	6.5%
Years included (omit terminal half-cycles)	full	1883 to 1923	1883 to 1936
The Major cycle	29.4%	25.5%	21.6%
Year included (omit terminal half-cycles)	full	1895 to 1917	1895 to 1930

Table R. PORTLAND CEMENT PRODUCTION IN THE UNITED STATES, 1880 TO 1939

Two stages of smoothing

Sources: Mineral Resources of the United States; Yearbooks of Commerce.

Part a (two pages)

1	2a or 4a	12a	12b	5	13	20a	20b	24
Year	Portland Cement Production (1000's barrels)	Smoothing Line B (by inspection)		Phase Point (short cycle)	Phase Point (major cycle)	Second Approximation to SL M		Final SL M (1000's bbls.)
		(thousands barrels)	logarithm			thousands barrels	logarithm	
1880	42	(52)						(30)
81	60	(61)		r_1				(38)
82	85	(75)		p_1				(49)
83	90	90		f_1				(64)
84	100	108		t_1 r_2				(81)
85	150	133		p_2 f_2				(105)
86	150	170		t_2 r_3				(135)
87	250	211		p_3				(172)
88	250	250		f_3				(222)
89	300	300			f_1			(290)
1890	335	355	2.55023	t_3 r_4		370	2.56820	(365)
91	455	430	2.63347	p_4		475	2.67669	(475)
92	547	515	2.71181			600	2.77815	(610)
93	591	610	2.78533	f_4 t_4		785	2.89487	(800)
94	799	770	2.88649	r_5 p_5 f_5	t_1	1030	3.01284	(1050)
95	990	1040	3.01703	t_5		1350	3.13033	1450
96	1543	1550	3.19033	r_6		1800	3.25527	2000
97	2678	2450	3.38917	p_6		2380	3.37658	2700
98	3692	3700	3.56820		r_1	3150	3.49831	3750
99	5652	5700	3.75580			4200	3.62325	5100
1900	8482	8500	3.92942	f_6		5600	3.74819	7000
01	12711	12700	4.10380			7500	3.87506	9200
02	17231	17300	4.23805			9600	3.98227	12500
03	22343	22400	4.35025			12300	4.08990	16000
04	26506	28500	4.45484	t_6		15400	4.18752	20300
05	35247	35300	4.54778	r_7		18800	4.27416	25400
06	46463	42000	4.62325	p_7		23000	4.36173	31300
1907	48783	49000	4.69020	f_7		28500	4.45484	37000

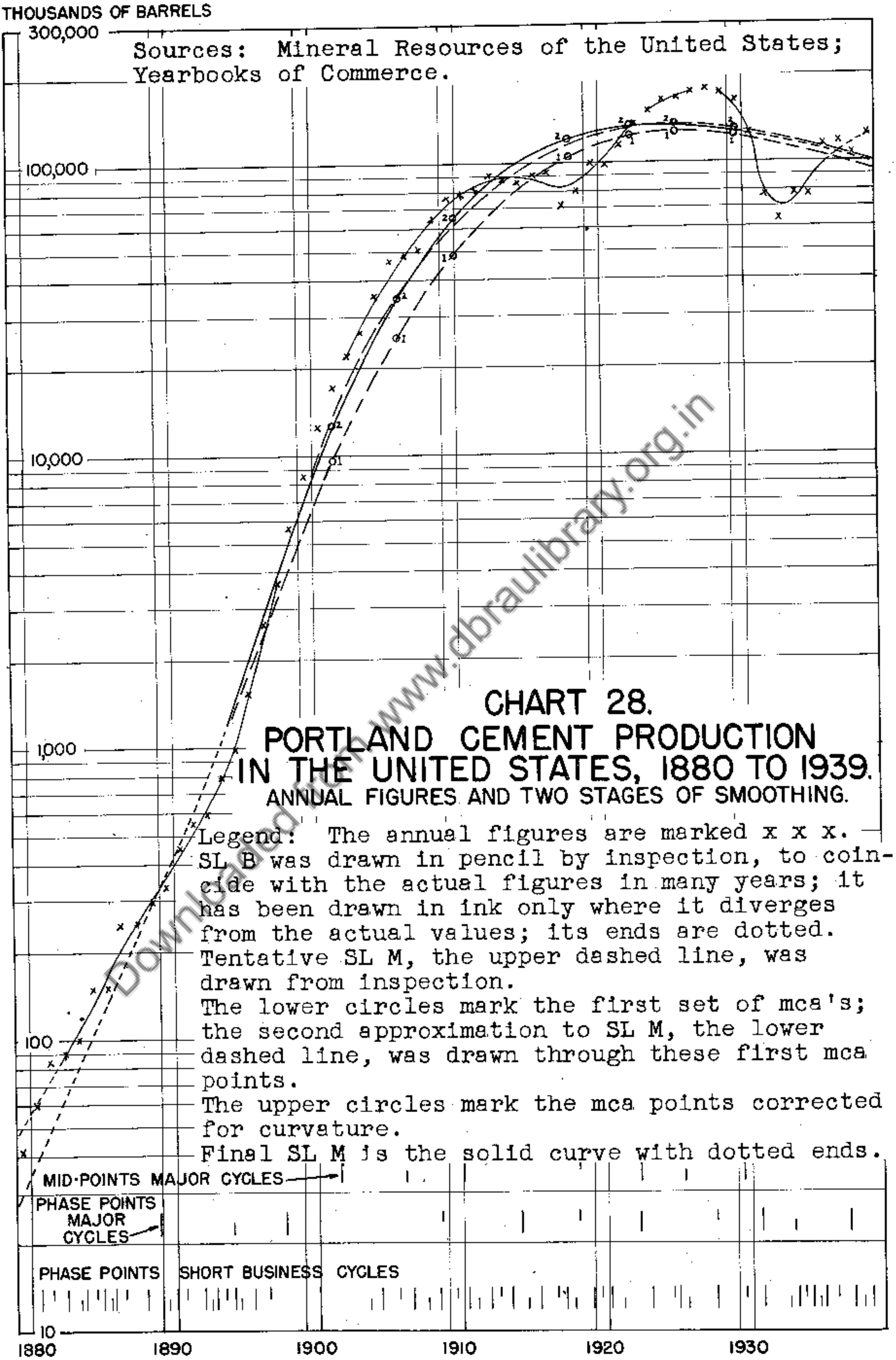
Table R, Part a (concluded) Portland Cement Production in the United States

1	2a or 4a	12a	12b	5	13	20a	20b	24
Year	Portland Cement Production (1000's barrels)	SL B Actual (thousands barrels)	SL B log	Phase Point	Phase Point (major cycle)	Second Approx. thousands barrels	Log of Second Approx.	SL M
1908	51073	56500	4.75205	t_7		33800	4.52892	45000
09	64991	65000	4.81291	r_8	p_1	39700	4.59879	53000
1910	76550	72500	4.86034	p8		46500	4.66745	61000
11	78529	80000	4.90309	f_8 t_8		52500	4.72016	69000
12	82438	85000	4.92942	r_9		59000	4.77085	77500
13	92097	88000	4.94448	p_9		66500	4.82282	85500
14	88230	90000	4.95424	f_9	f_2	74000	4.86923	93500
15	85915	90000	4.95424	t_9		81000	4.90848	101000
16	91521	88500	4.49694	r_{10}		88000	4.94448	108000
17	92814	84500	4.92686	p_{10}		95000	4.97772	114000
18	71082	82000	4.91381	f_{10} t_{10}		102000	5.00860	119000
19	80778	85500	4.93197	r_{11}	t_2	107000	5.02938	124000
1920	100023	93000	4.96848	p_{11}		112000	5.04922	127000
21	98842	105000	5.02119	f_{11}		117000	5.06819	130000
22	114790	121000	5.08278	t_{11}		122000	5.08636	132000
23	137460	137000	5.13672	r_{12}	r_2	125000	5.09691	133000
24	150777	151000	5.17898			127000	5.10380	133000
25	163388	161000	5.20683	p_{12} f_{12}		128000	5.10721	133000
26	166635	170000	5.23045	t_{12}		128000	5.10721	132000
27	175330	175000	5.24304			128000	5.10721	131000
28	178509	178000	5.25042	r_{13}	p_2	127000	5.10380	130000
29	127856	173000	5.23805			125000	5.09691	129000
1930	162989	156000	5.19312	p_{13}		124000	5.09342	127000
31	126671	127000	5.10380	f_{13}	f_3	121000	5.08278	(124000)
32	77198	82000	4.91381			117000	5.06819	(121000)
33	63984	70000	4.84510	t_{13} r_{14}	t_3	114000	5.05690	(118000)
34	78419	73000	4.86332	p_{14}		111000	5.04532	(115000)
35	77748	86000	4.93450	f_{14} t_{14}		108000	5.03342	(112000)
36	114469	99000	4.99564	r_{15} p_{15}		104000	5.01703	(109000)
37	118075	(10700)	5.02938	f_{15}	r_3	101000	5.00432	(105000)
38	107178	(116000)		t_{15}				(102000)
1939	124698	(122000)		r_{16}				(99000)

Table R. PORTLAND CEMENT PRODUCTION

Part b part of the calculations for the second stage of smoothing, including correction for curvature.

14 Cycle	15 Yearly Figures Included in the Cycle Begin	16 Middle	17 End	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (first application - 1000's barrels)	20a (to Table Ra) Second Approximation to SL M	20b Log of Second Approximation	21a Moving Total of Logs of Second Approximation	21b mca of Logs of Second Approx.	22 Difference in Logs (column 19 b minus column 21 b)	23a Adjusted mca of Logs	23b Adjusted Moving Cyclical Geometric Mean (1000's barrels)	24 (to Table Ra) Final SL M	16 Year
$f_{1,2}$	1890	1902	1914	25	99.58205	3.98328	9622			96.66640	3.86666	.11662	4.09990	12587		1902
$t_{1,2}$	1895	1906-07	1918	24	105.75658	4.40652	25499			102.57494	4.27396	.13256	4.53908	34601		1906-07
$r_{1,2}$	1899	1910-11	1922	24	112.59627	4.69151	49149			109.54756	4.56448	.12703	4.81854	65848		1910-11
$p_{1,2}$	1909	1918-19	1928	20	100.39720	5.01986	104680			99.14787	4.95739	.06247	5.08233	120870		1918-19
$f_{2,3}$	1915	1923	1931	17	86.52769	5.08986	122990			85.97168	5.05716	.03270	5.12256	132610		1923
$t_{2,3}$	1919	1926	1933	15	76.54474	5.10298	126760			76.25747	5.08383	.01915	5.12213	132470		1926
$r_{2,3}$	1923	1930	1937	15	76.36316	5.09088	123280			76.12446	5.07496	.01592	5.10680	127880		1930



Sources: Mineral Resources of the United States; Yearbooks of Commerce.

CHART 28.

PORTLAND CEMENT PRODUCTION IN THE UNITED STATES, 1880 TO 1939.

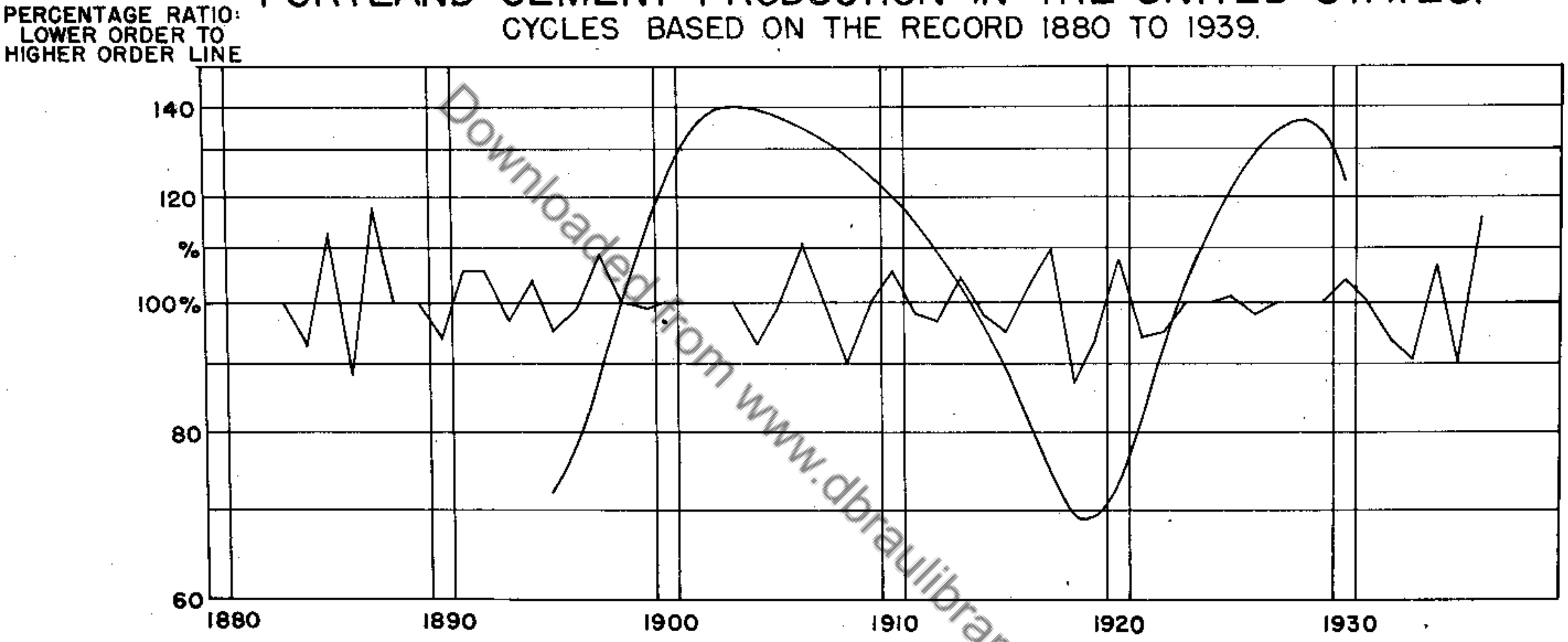
ANNUAL FIGURES AND TWO STAGES OF SMOOTHING.

Legend: The annual figures are marked x x x.
SL B was drawn in pencil by inspection, to coincide with the actual figures in many years; it has been drawn in ink only where it diverges from the actual values; its ends are dotted.
Tentative SL M, the upper dashed line, was drawn from inspection.
The lower circles mark the first set of mca's; the second approximation to SL M, the lower dashed line, was drawn through these first mca points.
The upper circles mark the mca points corrected for curvature.
Final SL M is the solid curve with dotted ends.

CHART 29.

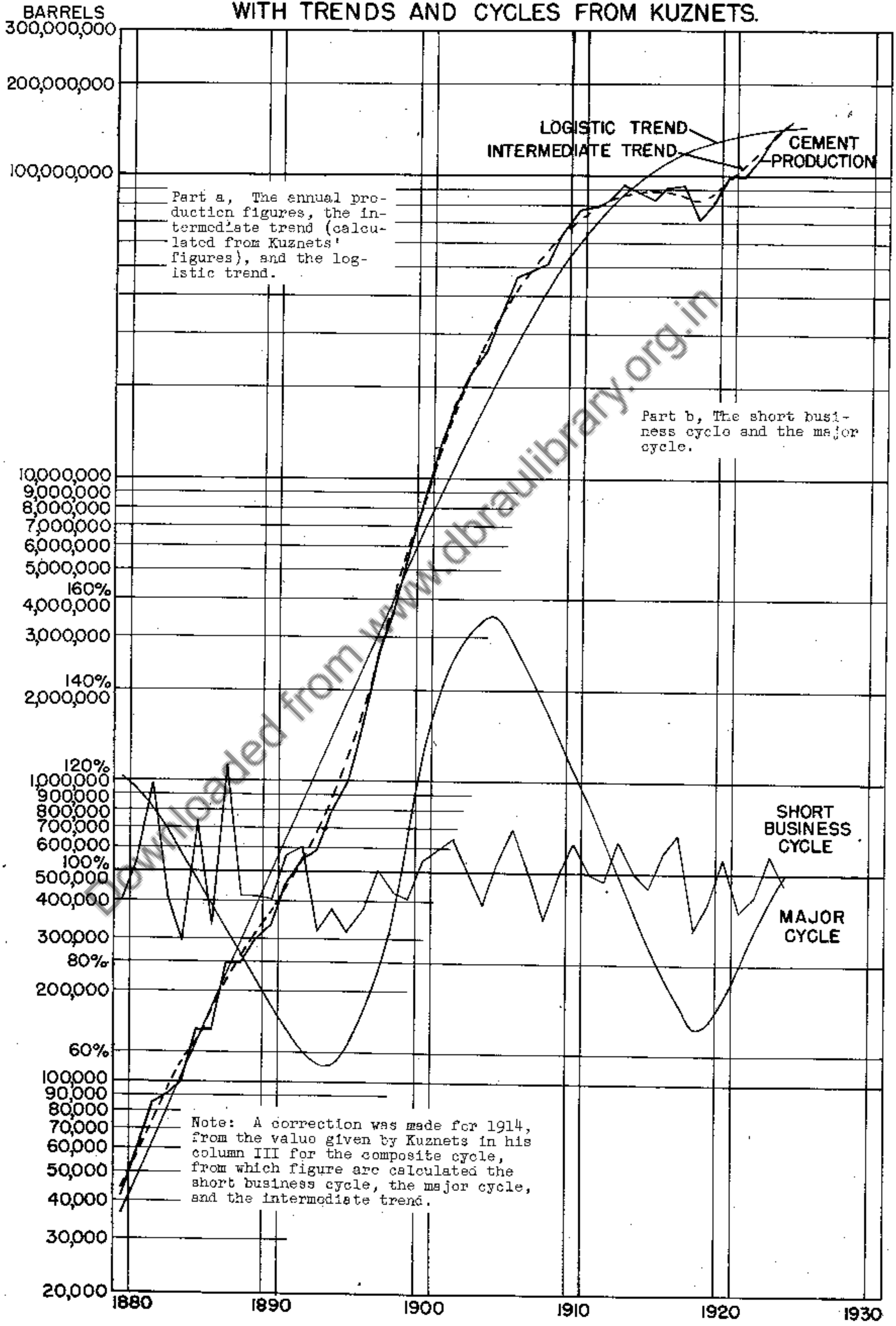
PORTLAND CEMENT PRODUCTION IN THE UNITED STATES.

CYCLES BASED ON THE RECORD 1880 TO 1939.



Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual production to SL B.
The curve shows the major cycle, the ratio of SL B to·SL M.

CHART 30.
PORTLAND CEMENT PRODUCTION IN THE UNITED STATES, 1880 TO 1924.
WITH TRENDS AND CYCLES FROM KUZNETS.



Part a, The annual production figures, the intermediate trend (calculated from Kuznets' figures), and the logistic trend.

Part b, The short business cycle and the major cycle.

Note: A correction was made for 1914, from the value given by Kuznets in his column III for the composite cycle, from which figure are calculated the short business cycle, the major cycle, and the intermediate trend.

Table S. PORTLAND CEMENT PRODUCTION IN THE UNITED STATES

Calculation of standard measures of the two orders of cycles.

Based on the record 1880 to 1939. Three pages.

1 Year	2a or 4a Production of Portland Cement	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of barrels)									
1880	42				The short business cycle					1880
1881	60									81
82	85									82
83	90	90		100	0	0				83
84	100	108		93	- 7	49				84
85	150	133		113	+13	169				85
86	150	170		88	-12	144				86
87	250	211		118	+18	324				87
88	250	250		100	0	0				88
89	300	300		100	0	0				89
1890	335	355		94	- 6	36		The major cycle		1890
91	455	430		106	+ 6	36				91
92	547	515		106	+ 6	36				92
93	591	610		97	- 3	9				93
94	799	770		104	+ 4	16				94
95	990	1040	1450	95	- 5	25	72	-28	784	95
96	1543	1550	2000	99	- 1	1	77	-23	529	96
97	2678	2450	2700	109	+ 9	81	91	- 9	81	97
98	3692	3700	3750	100	0	0	99	- 1	1	98
99	5652	5700	5100	99	- 1	1	112	+12	144	99
1900	8482	8500	7000	100	0	0	121	+21	441	1900
01	12711	12700	9300	100	0	0	136	+36	1296	01
02	17231	17300	12300	100	0	0	141	+41	1681	02
03	22343	22400	16000	100	0	0	140	+40	1600	03
04	26506	28500	20300	93	- 7	49	140	+40	1600	04
05	35247	35300	25400	100	0	0	139	+39	1521	05
06	46463	42000	31300	111	+11	121	134	+34	1156	06
1907	48783	49000	37000	100	0	0	132	+32	1024	1907

Table S (continued) Portland Cement Production.

1 Year	2a or 4a Production of Portland Cement	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M %	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of barrels)									
1908	51073	56500	45000	90	-10	100	126	+26	676	1908
09	64991	65000	53000	100	0	0	123	+23	529	09
1910	76550	72500	61000	106	+ 6	36	119	+19	361	1910
11	78529	80000	69000	98	- 2	4	116	+16	256	11
12	82438	85000	77500	97	- 3	9	110	+10	100	12
13	92097	88000	85500	105	+ 5	25	103	+ 3	9	13
14	88230	90000	93500	98	- 2	4	96	- 4	16	14
15	88915	90000	101000	95	- 5	25	89	-11	121	15
16	91521	88500	108000	103	+ 3	9	82	-18	324	16
17	92814	84500	114000	110	+10	100	74	-26	676	17
18	71082	82000	119000	87	-13	169	69	-31	961	18
19	80778	85500	124000	94	- 6	36	69	-31	961	19
1920	100023	93000	127000	108	+ 8	64	73	-27	729	1920
21	98842	105000	130000	94	- 6	36	81	-19	361	21
22	114790	121000	132000	95	- 5	25	92	- 8	64	22
23	137460	137000	133000	100	0	0	103	+ 3	9	23
24	150777	151000	133000	100	0	0	114	+14	196	24
25	163388	161000	133000	101	+ 1	1	121	+21	441	25
26	166635	170000	132000	98	- 2	4	129	+29	841	26
27	175330	175000	131000	100	0	0	134	+34	1156	27
28	178509	178000	130000	100	0	0	137	+37	1369	28
29	172856	173000	129000	100	0	0	134	+34	1156	29
1930	162989	156000	127000	104	+ 4	16	123	+23	529	1930
31	126671	127000		100	0	0				31
32	77198	82000		94	- 6	36				32
33	63984	70000		91	- 9	81				33
34	78419	73000		107	+ 7	49				34
35	77748	86000		90	-10	100				35
36	114469	99000		116	+16	256				36
37	118075									37
38	107178									38
1939	124698									1939

The short business cycle.

Column 27, at 1923: end of the short period for calculation of sd.

Column 32, at 1917: end of the short period for calculation of sd.

$\Sigma(d^2) = 2282$

$sd = \sqrt{\frac{2282}{54}} = 6.5\%$, in the short business cycle. (the full period)

$\Sigma(d^2) = 23699$

$sd = \sqrt{\frac{23699}{51}} = 21.6\%$, in the major cycle. (the full period)

(Table S is concluded on the next page.)

Table S (concluded) Portland Cement Production

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles[28]
of actual about SL B (the short business cycle)

Cycle Number	Lengths in Years rp	pf	ft	tr
1	.8	1.0	1.0	.3
2	.7	.5	.5	.3
3	.7	1.5	1.5	.5
4	.6	1.2	.5	.4
5	.6	.6	.4	1.0
6	1.0	3.5	3.5	.8
7	1.2	1.0	1.0	.8
8	1.2	.8	.7	.9
9	.6	.8	1.2	.8
10	1.2	.6	.6	1.2
11	.6	.7	.8	2.0
12	1.5	.4	.6	2.0
13	2.0	1.2	1.8	.5
14	.5	.5	.5	.3
15	.9	1.1	.6	.6
Average Length	.95 yr.	1.03 yr.	1.01 yr.	.83 yr.

Total length of typical short business cycle 3.8 yrs.

of SL B about SL M (the major cycle)[33]

1			5.0	3.8
2	10.7	5.5	4.0	4.2
3	5.3	3.0	2.3	3.7
Average Length	8.00 yrs.	4.25 yrs.	3.77 yrs.	3.90 yrs.

Total length of typical major cycle 19.9 yrs.

Percentage Deviations at Peak and Trough
of actual from SL B (the short business cycle)[29]

At Peak Year	Deviation	At Trough Year	Deviation
		1884	7%
1885	13%	86	12
87	18	90	6
91	6	93	3
94	4	95	5
97	9	1904	7
1906	11	08	10
10	6	11-12	3
13	5	15	5
17	10	18	13
20	8	21-22	6
25	1	26	2
30	4	33	9
34	7	35	10
36	16		
Average deviation in the short business cycle	12.4% at peak		8.8% at trough

of SL B from SL M (the major cycle)[34]

		1894-95	28%
1902	41%	1918-19	31
28	37		
Average deviation in the major cycle	39 % at peak		29.5% at trough

A Comparison of Kuznets' Trend Line with Smoothing Line M

Period Reported	Center Date of Period	Trend by Kuznets' first equation (in thousands of short tons)	SL M
1838 - 40	Dec. 31, 1838	764	
1841 - 45	June 30, 1843	1061	
1846 - 50	June 30, 1848	1421	
1851 - 55	June 30, 1853	1828	(1950)
1856 - 60	June 30, 1858	2250	(2240)
1861 - 65	June 30, 1863	2651	(2570)
1866 - 70	June 30, 1868	3000	(3020)
Year			
1873		3284	(3300)
1878		3493	3570
1880		3570	3630
		Trend by Kuznets' second equation	
1881		3542	3640
1885		3411	3570
1890		3200	3180
1895		2934	2770
1900		2630	2400
1905		2270	2000
1910		1900	1650
1915		1537	1400
1920		1203	1450
1925			1760
1930			2300
1935			3150

Kuznets' equation for the period 1837 to 1880

$$y = \frac{4000}{1+10^{(1.10506 - 0.73596x)}}$$

x in units of 20 years; origin at 1825.

Kuznets' equation for the period 1881 to 1922

$$y = 4000 - \frac{4000}{1+10^{(1.24758 - 0.64553x)}}$$

x in units of 20 years; origin at 1870.

IN the case of the freight carried by the Erie Canal, Kuznets fitted separate trend lines to periods that seemed to be marked by different directions of change in the value of the variable. This device of breaking a series into parts and fitting trend lines separately to those parts, does a certain violence to the idea of continuity of movement in a trend. The operator, in following such a procedure, seems to have admitted that there is not homogeneity of the underlying forces throughout the period. But when he breaks the period into two parts, and fits a trend to each part by a total process, he assumes homogeneity <u>within</u> each of those two parts.

KUZNETS' first equation, fitted to the period ending in 1880, gives a trend line which depicts growth. The second equation, fitted to the data beginning in 1881, gives a trend line which depicts a decline in the value of the variable. It will be noted that Kuznets' second period ends with 1922. He omitted the data for 1923, 1924 and 1925. Had these been plotted, they would not have gone well with his second trend line. Should he now undertake to fit trend lines to this series, based upon data running to 1940 and later, he might be tempted to break the series into three parts instead of two. His first point of division, 1880-81, need not be challenged, but his second point of division would now probably be set at 1918.

THE process of fitting the smoothing lines, it will be seen, was really begun about 1870. Prior to that time, annual data were not available, so a simple free-hand technique was employed without any kind of objective check. Yet even that early portion of SL M checks rather well with Kuznets' fitted trend.

THROUGH the succeeding 12 years of Kuznets' first equation, the agreement is quite close, with SL M slightly higher. This little difference is probably because SL M moves to a peak, whereas Kuznets' curve flattens off to a plateau.

IN the period of Kuznets' second equation, 1881 to 1922, SL M is higher at the initial dates for the reason just noted, namely that SL M moves from a peak there, but by 1890 the two lines intersect, and from that date to 1915, SL M runs below Kuznets' line. A reason for the 1915 intersection is that SL M has already begun to feel the increase in the amount of traffic on the Canal after 1917. SL M rises to a value in 1920 which departs quite markedly from the continuing decline in Kuznets' trend line.

NO formal step was taken, in determining the location of SL M, to correct the line for curvature. Possibly such correction would have given slightly higher values from 1880 to 1890, and would thereby have increased the discrepancy from Kuznets' lines. Possibly also such correction would have given lower values of SL M from 1910 to 1925, which would have tended to reduce the discrepancy from Kuznets' line. But the changes so effected would probably not have been great.

IN the case of this series, Kuznets does not make his usual analysis into major cycle and short business cycle.

Values of the standard deviation, by the several calculations:

	Kuznets' figures not available	Figures secured by the method of smoothing by stages	
		based on period 1870 to 1922	based on period 1870 .to 1938
The short business cycle		6.4%	6.4%
Years included (omit terminal half-cycles)		1872 to 1920	1872 to 1936
The major cycle		10.8%	17.5%
Years included (omit terminal half-cycles)		1878 to 1912	1878 to 1925

Table T. ERIE CANAL FREIGHT MOVED, 1851 TO 1939

Two stages of smoothing.

Source: Statistical Abstract for the United States.

Part a (four pages)

1 Year	2a or 4a Freight Moved	2b or 4b Freight Moved	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle	8 Yearly Figures Included in the Cycle	9 Yearly Figures Included in the Cycle	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Freight Moved (1000's tons)	12a Smoothing Line B	12b Smoothing Line B	13 Phase Point (major cycle)	24 SL M (thousands short tons)	1 Year
	thousands short tons	logarithm			Begin	Middle	End					thousands short tons	logarithm			
1837-40	772															1837-40
1841-45	884															1841-45
1846-50	1557															1846-50
1851-55	2142	Yearly averages														1851-55
1856-60	1890															1856-60
1861-65	2744															1861-65
1866-70	3018															1866-70
1870	3083	3.48897										(3220)	3.50786		(3100)	1870
71	3581	3.55400	r_1 p_1	$r_{1,2}$	1871	1871-72	1872	2	7.10582	3.55291	3572	(3470)	3.54033		(3150)	71
72	3563	3.55182	f_1 t_1	$p_{1,2}$	1871	1872	1873	3	10.66548	3.55516	3591	3600	3.55630		(3220)	72
73	3603	3.55666	r_2 p_2	$f_{1,2}$	1872	1873	1874	3	10.59942	3.53314	3413	3420	3.53403	f_1	(3300)	73
74	3097	3.49094		$t_{1,2}$ $r_{2,3}$	1872 1873	1874 1874-75	1876 1876	5 4	17.42802 13.87620	3.48560 3.46905	3059 2945	3050	3.48430		(3350)	74
75	2787	3.44514	f_2	$p_{2,3}$	1873	1875	1877	5	17.38862	3.47772	3004	2830	3.45179		(3420)	75
76	2418	3.38346	t_2	$f_{2,3}$	1875	1876	1878	4	13.89840	3.47460	2983	2810	3.44871	t_1	(3480)	76
77	3254	3.51242	r_3 p_3									3070	3.48714		(3530)	77
78	3609	3.55739	f_3	$t_{2,3}$ $r_{3,4}$	1876 1877	1877-78 1878	1879 1879	4 3	14.03533 10.65187	3.50883 3.55062	3227 3553	3600	3.55630	r_1	3570	78
79	3820	3.58206	t_3	$p_{3,4}$ $f_{3,4}$	1877 1878	1878-79 1879	1880 1880	4 3	14.31548 10.80206	3.57887 3.60039	3792 3985	4030	3.60531		3610	79
1880	4609	3.66361	r_4 p_4	$t_{3,4}$	1879	1880	1881	3	10.80185	3.60062	3987	4100	3.61278	p_1	3630	1880

Table T (continued) Erie Canal Freight Moved.

1 Year	2a or 4a Freight Moved	2b or 4b	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle	8	9	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean of Freight Moved (1000's tons)	12a Smoothing Line B	12b	13 Phase Point (major cycle)	24 SL M (thousands short tons)	1 Year
	thousands short tons	logarithm			Begin	Middle	End					thousands short tons	logarithm			
1881	3599	3.55618	f_4 t_4	$r_{4,5}$ $p_{4,5}$	1880 1880	1880-81 1881	1881 1882	2 3	7.21979 10.78829	3.60989 3.59676	4073 3951	3830	3.58320	f_2	3640	1881
82	3694	3.56750	r_5 p_5	$f_{4,5}$	1881	1882	1883	3	10.67841	3.55947	3626	3600	3.55630		3630	82
83	3587	3.55473		$t_{4,5}$	1881	1883	1884	4	14.20861	3.55215	3566	3500	3.54407	t_2	3620	83
84	3390	3.53020	f_5	$r_{5,6}$ $p_{5,6}$	1882 1883	1884 1884-85	1885 1886	4 4	14.15867 14.17198	3.53987 3.54299	3466 3491	3430	3.53908		3600	84
85	3208	3.50624	t_5 r_6									3450	3.54283		3570	85
86	3809	3.58081		$f_{5,6}$ $t_{5,6}$	1884 1885	1885-86 1886	1887 1888	4 4	14.20169 14.19289	3.55042 3.54822	3552 3534	3550	3.54777	r_2	3500	86
87	3841	3.58444	p_6	$r_{6,7}$	1886	1887	1888	3	10.68665	3.56222	3649	3600	3.54900		3440	87
88	3322	3.52140	f_6 t_6	$p_{6,7}$	1887	1888	1889	3	10.67198	3.55733	3609	3530	3.54407		3350	88
89	3674	3.56514	r_7 p_7	$f_{6,7}$ $t_{6,7}$	1888 1888	1889 1889-90	1890 1891	3 4	10.60558 14.09666	3.53519 3.52417	3429 3343	3420	3.53275	p_2	3270	89
1890	3304	3.51904	f_7									3300	3.51851		3180	1890
91	3098	3.49108	t_7	$r_{7,8}$ $p_{7,8}$	1889 1889	1890-91 1891	1892 1893	4 5	14.04933 17.55934	3.51233 3.51187	3253 3250	3200	3.50515		3100	91
92	2979	3.47407	r_8	$f_{7,8}$	1890	1892	1894	5	17.49168	3.49834	3150	3080	3.48855		3000	92
93	3236	3.51001		$t_{7,8}$	1891	1893	1895	5	17.34482	3.46896	2944	2970	3.47276	f_3	2940	93
94	3144	3.49748	p_8 f_8	$r_{8,9}$	1893	1894	1895	3	10.37967	3.45989	2883	2820	3.45025		2850	94
95	2356	3.37218	t_8	$p_{8,9}$	1894	1895	1896	3	10.30773	3.43591	2728	2690	3.42975		2770	95
96	2742	3.43807	r_9 p_9	$f_{8,9}$ $t_{8,9}$	1895 1895	1896 1896-97	1897 1898	3 4	10.22271 13.59155	3.40757 3.39789	2556 2500	2600	3.41497		2690	96
97	2585	3.41246		$r_{9,10}$	1896	1897	1898	3	10.21937	3.40646	2550	2500	3.39794		2620	97
98	2338	3.36884	f_9 t_9	$p_{9,10}$	1896	1898	1899	4	13.60301	3.40100	2518	2400	3.38021		2550	98
99	2419	3.38364	r_{10} p_{10}	$f_{9,10}$ $t_{9,10}$	1898 1898	1898-99 1899	1899 1900	2 3	6.75248 10.08411	3.37624 3.36127	2378 2298	2320	3.36549	t_3	2480	99
1900	2146	3.33163	f_{10} t_{10}	$r_{10,11}$ $p_{10,11}$	1899 1899	1899-00 1900	1900 1901	2 3	6.71527 10.06880	3.35763 3.35627	2278 2271	2250	3.35218		2400	1900
01	2257	3.35353	r_{11} p_{11}	$f_{10,11}$ $t_{10,11}$	1900 1900	1900-01 1901	1901 1902	2 3	6.68516 10.00862	3.34258 3.33621	2201 2169	2200	3.34242		2330	01

(Table T, part a, is continued on next page.)

Table T, Part a, (continued) Erie Canal Freight Moved

1 Year	2a or 4a Freight Moved	2b or 4b	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle	8	9	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geometric Mean Freight (1000's tons)	12a Smoothing Line B	12b	13 Phase Point (major cycle)	24 SL M 1000's tons	1 Year
	thousands tons	logarithm			Begin	Middle	End					thousands tons	logarithm			
			f_{11}	$r_{11,12}$	1901	1901	1902	2	6.67699	3.33849	2180					
1902	2106	3.32346	t_{11}	$p_{11,12}$	1901	1902	1903	3	10.05973	3.35324	2255	2170	3.33646		2250	1902
			r_{12}	$f_{11,12}$	1902	1902-03	1903	2	6.70620	3.35310	2255					
1903	2414	3.38274	p_{12}	$t_{11,12}$	1902	1903	1904	3	9.99534	3.33178	2147	2150	3.33244	r_3	2170	03
1904	1946	3.28914	f_{12}	$r_{12,13}$	1903	1904	1905	3	9.97291	3.32764	2126	2160	3.33445		2090	04
05	2000	3.30103	t_{12}	$p_{12,13}$	1903	1905	1906	4	13.35040	3.33760	2176	2170	3.33646		2000	05
06	2385	3.37749	r_{13}	$f_{12,13}$	1904	1906	1908	5	16.68862	3.33772	2176	2190	3.34044		1930	06
07	2416	3.38310	p_{13}	$t_{12,13}$	1905	1907	1909	5	16.70719	3.34144	2195	2200	3.34242		1840	07
08	2177	3.33786	f_{13}	$r_{13,14}$	1906	1908	1910	5	16.71216	3.34243	2200	2170	3.33646		1780	08
09	2031	3.30771	t_{13}	$p_{13,14}$	1907	1909	1911	5	16.64259	3.32852	2131	2130	3.32838		1710	09
1910	2023	3.30600		$f_{13,14}$	1908	1910	1911	4	13.25949	3.31487	2065	2070	3.31597	p_3	1650	1910
			r_{14}	$t_{13,14}$	1909	1910-11	1912	4	13.17569	3.29392	1968					
11	2032	3.30792	p_{14}	$r_{14,15}$	1911	1911	1912	2	6.56198	3.28099	1910	1970	3.29447		1580	11
12	1795	3.25406	f_{14} t_{14}	$p_{14,15}$	1911	1912	1913	3	9.81435	3.27145	1868	1850	3.26711		1520	12
			r_{15}	$f_{14,15}$	1912	1913	1913	2	6.50643	3.25322	1792					
13	1788	3.25237	p_{15}	$t_{14,15}$	1912	1913	1914	3	9.64061	3.21354	1635	1670	3.22272	f_4	1480	13
			f_{15}	$r_{15,16}$	1913	1913	1914	2	6.38655	3.19328	1561					
14	1362	3.13418	t_{15}	$p_{15,16}$	1913	1914	1915	3	9.44913	3.14971	1412	1350	3.13033		1430	14
15	1155	3.06258	r_{16} p_{16}	$f_{15,16}$	1914	1915	1916	3	90.15960	3.05320	1130	1100	3.04139		1400	15
16	918	2.96284	f_{16}	$t_{15,16}$	1914	1916	1917	4	11.98890	2.99722	994	880	2.94448		1390	16
				$r_{16,17}$	1915	1916-17	1918	4	11.67885	2.91971	831					
17	675	2.82930		$p_{16,17}$	1915	1917	1919	5	14.60416	2.92083	833	750	2.87506		1380	17
1918	667	2.82413	t_{16}	$f_{16,17}$	1916	1918	1920	5	14.49146	2.89829	791	730	2.86332	t_4	1400	1918

Table T. is concluded on the next page.

Table T (concluded) Erie Canal Freight Moved

1 Year	2a or 4a Freight Moved thousands tons	2b or 4b Freight Moved logarithm	5 Phase Point (short cycle)	6 Cycle	7 Yearly Figures Included in the Cycle Begin	8 Yearly Figures Included in the Cycle Middle	9 Yearly Figures Included in the Cycle End	10 Length in Years	11a Moving Total of Logs	11b Moving Cyclical Average of Logs	11c Moving Cyclical Geom. Mean Freight (1000's tons)	12a Smoothing Line B thousands tons	12b Smoothing Line B logarithm	13 Phase Point (major cycle)	24 SL M 1000's tons	1 Year
19	842	2.92531	r_{17} p_{17}	$t_{16,17}$	1918	1919	1921	4	11.69671	2.92418	840	780	2.89210		1420	19
1920	891	2.94988	f_{17}	$r_{17,18}$	1919	1920	1921	3	8.87258	2.95753	907	910	2.95904		1450	1920
1921	994	2.99739	t_{17}	$p_{17,18}$ $f_{17,18}$	1919 1920	1921 1921-22	1922 1923	4 4	12.04431 12.33012	3.01108 3.08503	1026 1216	1130	3.05308		1490	21
22	1485	3.17173	r_{18} p_{18}									1380	3.13988		1530	22
23	1626	3.21112	f_{18}	$t_{17,18}$ $r_{18,19}$	1921 1922	1922-23 1923	1924 1924	4 3	12.60864 9.61125	3.15216 3.20375	1420 1599	1600	3.20412		1620	23
24	1692	3.22840	t_{18} r_{19}	$p_{18,19}$ $f_{18,19}$	1923 1924	1924 1924	1925 1925	3 2	9.72844 6.51732	3.24281 3.25866	1749 1814	1740	3.24055		1680	24
25	1945	3.28892	p_{19} f_{19}	$t_{18,19}$	1924	1925	1926	3	9.80400	3.26800	1854	1850	3.26717		1760	25
26	1935	3.28668	t_{19}	$r_{19,20}$ $p_{19,20}$	1925 1925	1926 1926-27	1927 1928	3 4	9.88693 13.29108	3.29564 3.32277	1975 2103	1980	3.29667		(1850)	26
27	2048	3.31133		$f_{19,20}$	1926	1927	1928	3	10.00216	3.33405	2158	2160	3.33445		(1960)	27
28	2536	3.40415	r_{20} p_{20}	$t_{19,20}$ $r_{20,21}$	1927 1928	1928 1928-29	1929 1929	3 2	10.09966 6.78833	3.36655 3.39416	2326 2478	2350	3.37107		(2060)	28
29	2422	3.38418	f_{20} t_{20}	$p_{20,21}$	1928	1929	1930	3	10.27178	3.42393	2654	2620	3.41830		(2180)	29
1930	3044	3.48345	r_{21} p_{21}	$f_{20,21}$ $t_{20,21}$	1929 1929	1930 1930-31	1931 1932	3 4	10.38324 13.88649	3.46108 3.47162	2891 2962	2900	3.46240		(2300)	1930
31	3278	3.51561	f_{21}	$r_{21,22}$	1930	1931	1932	3	10.50231	3.50077	3168	3110.	3.49276		(2450)	31
32	3186	3.50325	t_{21} r_{22}	$p_{21,22}$ $f_{21,22}$	1931 1932	1932 1932	1933 1933	3 2	10.57202 7.05641	3.52401 3.52820	3342 3374	3310	3.51983		(2600)	32
33	3574	3.55316	p_{22} f_{22}	$t_{21,22}$	1932	1933	1934	3	10.61811	3.53937	3462	3520	3.54654		(2780)	33
34	3645	3.56170	t_{22}	$r_{22,23}$	1933	1934	1935	3	10.70570	3.56857	3703	3700	3.56820		(2960)	34
35	3898	3.59084	r_{23}	$p_{22,23}$, $f_{22,23}$	1933 1934	1935 1935	1936 1937	4 4	14.33101 14.39829	3.58275 3.59956	3826 3977	3880	3.58883		(3150)	35
36	4220	3.62531	p_{23}	$t_{22,23}$	1934	1936	1938	5	17.92331	3.58468	3843	3940	3.59550		(3350)	36
37	4174	3.62055		$r_{23,24}$	1935	1937	1938	4	14.36161	3.59040	3894	(3850)	3.58546		(3560)	37
38	3349	3.52491	f_{23} t_{23}									(3620)	3.55871		(3800)	38
1939	3644	3.56158	r_{24}									(3300)	3.51851		(4050)	1939

Table T. ERIE CANAL FREIGHT MOVED

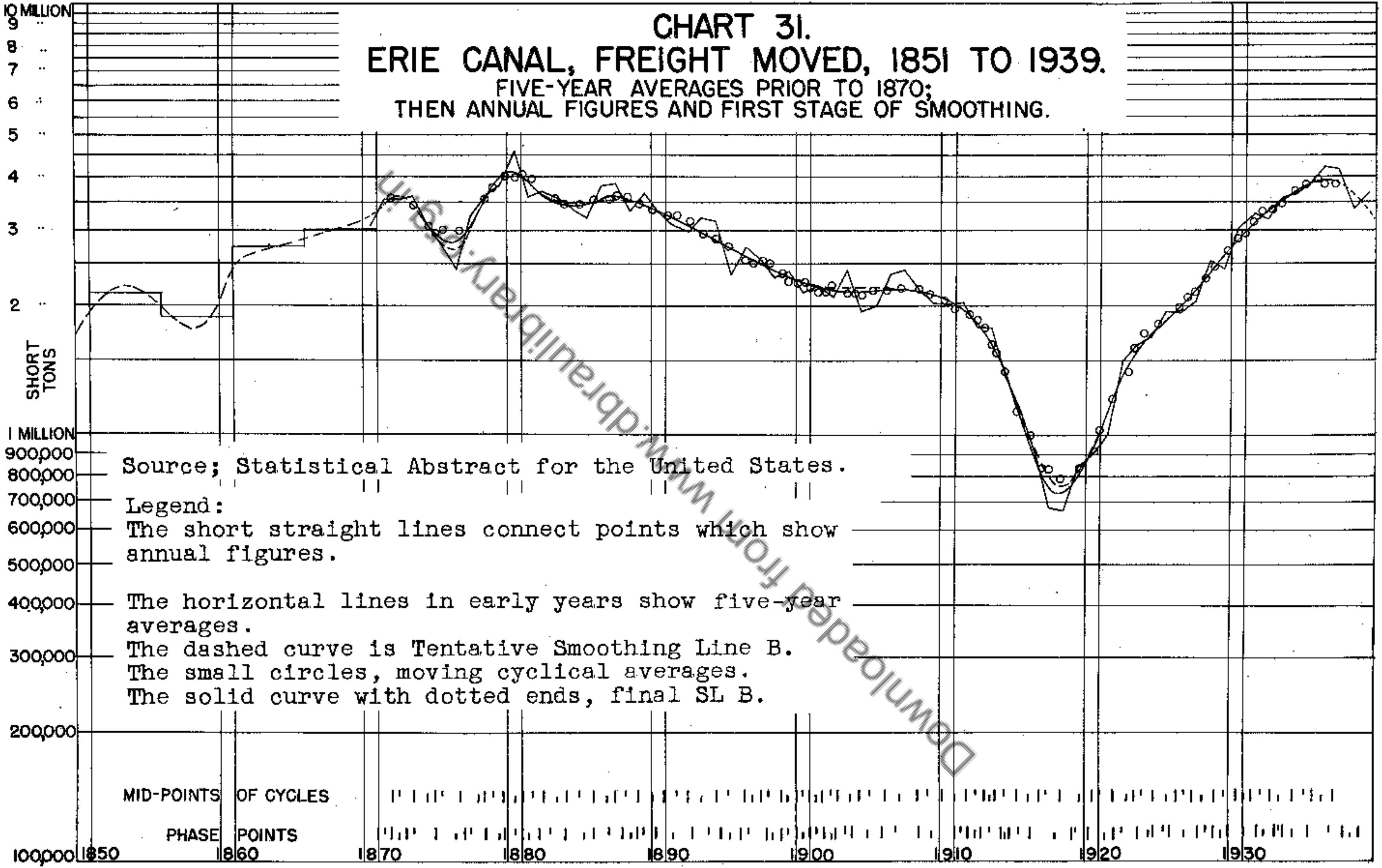
Part b part of the calculation for the second stage of smoothing.

14 Cycle	15 Yearly Figures Included in the Cycle Begin	16 Middle	17 End	18 Length in Years	19a Moving Total of Logs of SL B	19b Moving Cyclical Average of Logs	19c Moving Cyclical Geometric Mean (1000's barrels)
$f_{1,2}$	1874	1877	1881	8	28.22953	3.52869	3378
$t_{1,2}$	1876	1880	1883	8	28.39381	3.54923	3524
$r_{1,2}$	1878	1882	1885	8	28.53987	3.56748	3694
$p_{1,2}$	1880	1884-85	1889	10	35.55185	3.55519	3591
$f_{2,3}$	1882	1887-88	1893	12	42.34084	3.52840	3376
$t_{2,3}$	1884	1891-92	1899	16	55.67908	3.47994	3020
$r_{2,3}$	1886	1894	1902	17	58.62823	3.45171	2830
$p_{2,3}$	1889	1899	1909	21	71.33844	3.39707	2495
$f_{3,4}$	1894	1903	1913	20	66.92105	3.34605	2218
$t_{3,4}$	1900	1909	1918	19	61.24554	3.22345	1673
$r_{3,4}$	1903	1913-14	1924	22	69.79473	3.17249	1488
$p_{3,4}$	1910	1921	1933	24	77.15287	3.21554	1643
$f_{4,5}$	1914	1925-26	1937	24	78.39053	3.26627	1846

CHART 31.

ERIE CANAL, FREIGHT MOVED, 1851 TO 1939.

FIVE-YEAR AVERAGES PRIOR TO 1870;
THEN ANNUAL FIGURES AND FIRST STAGE OF SMOOTHING.



Source; Statistical Abstract for the United States.

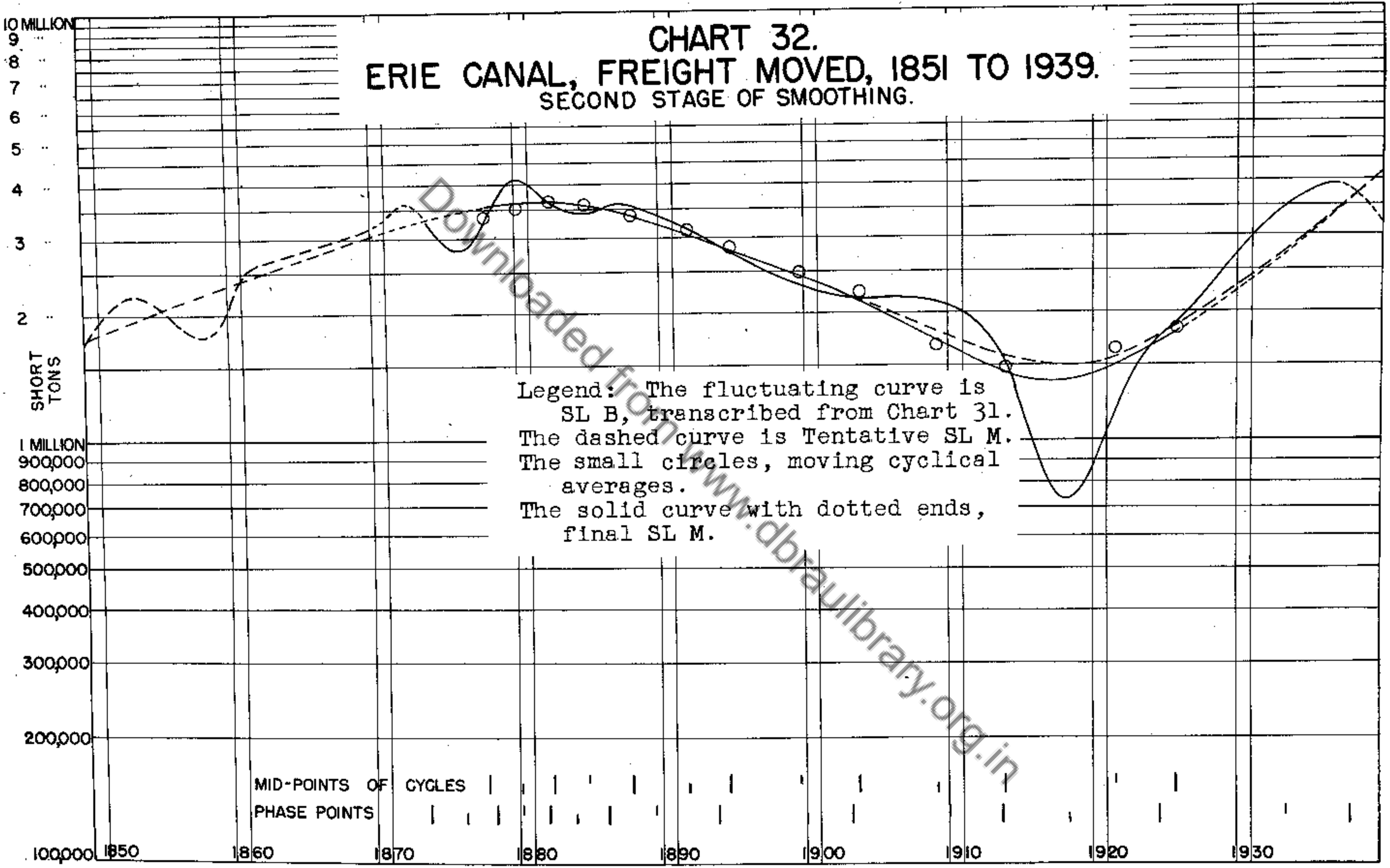
Legend:
The short straight lines connect points which show annual figures.

The horizontal lines in early years show five-year averages.
The dashed curve is Tentative Smoothing Line B.
The small circles, moving cyclical averages.
The solid curve with dotted ends, final SL B.

CHART 32.

ERIE CANAL, FREIGHT MOVED, 1851 TO 1939.

SECOND STAGE OF SMOOTHING.

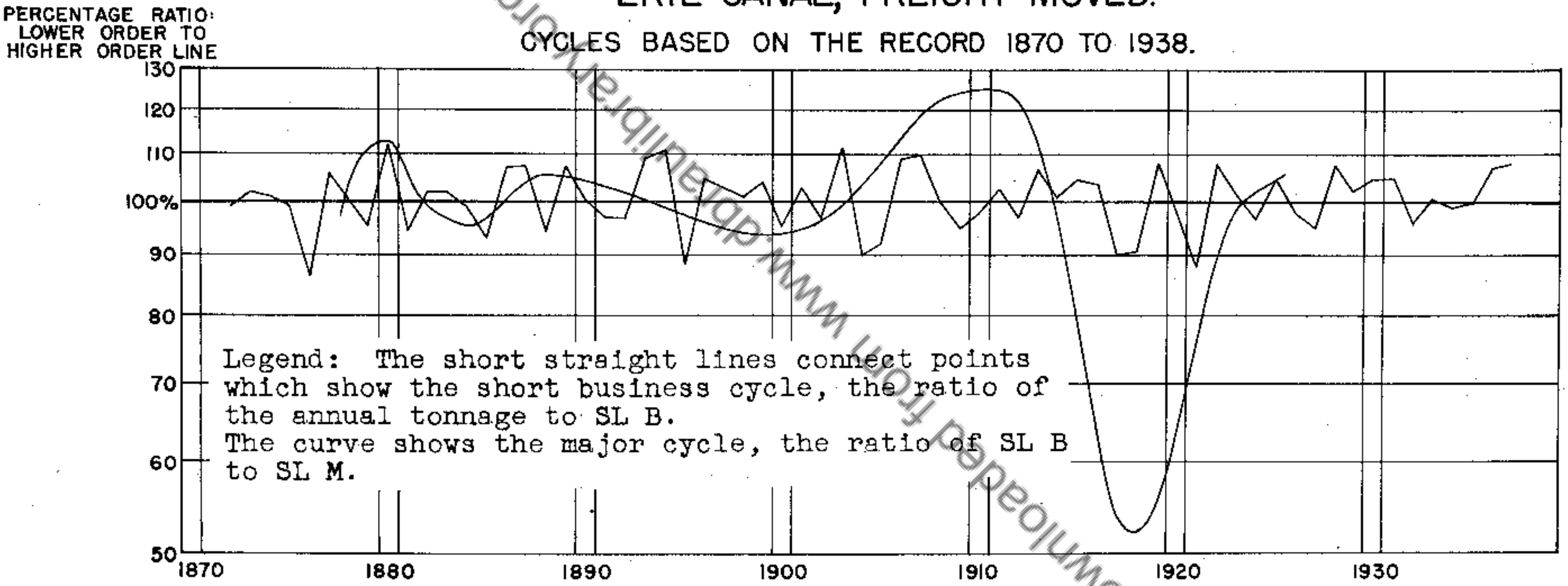


Legend: The fluctuating curve is SL B, transcribed from Chart 31.
The dashed curve is Tentative SL M.
The small circles, moving cyclical averages.
The solid curve with dotted ends, final SL M.

CHART 33.

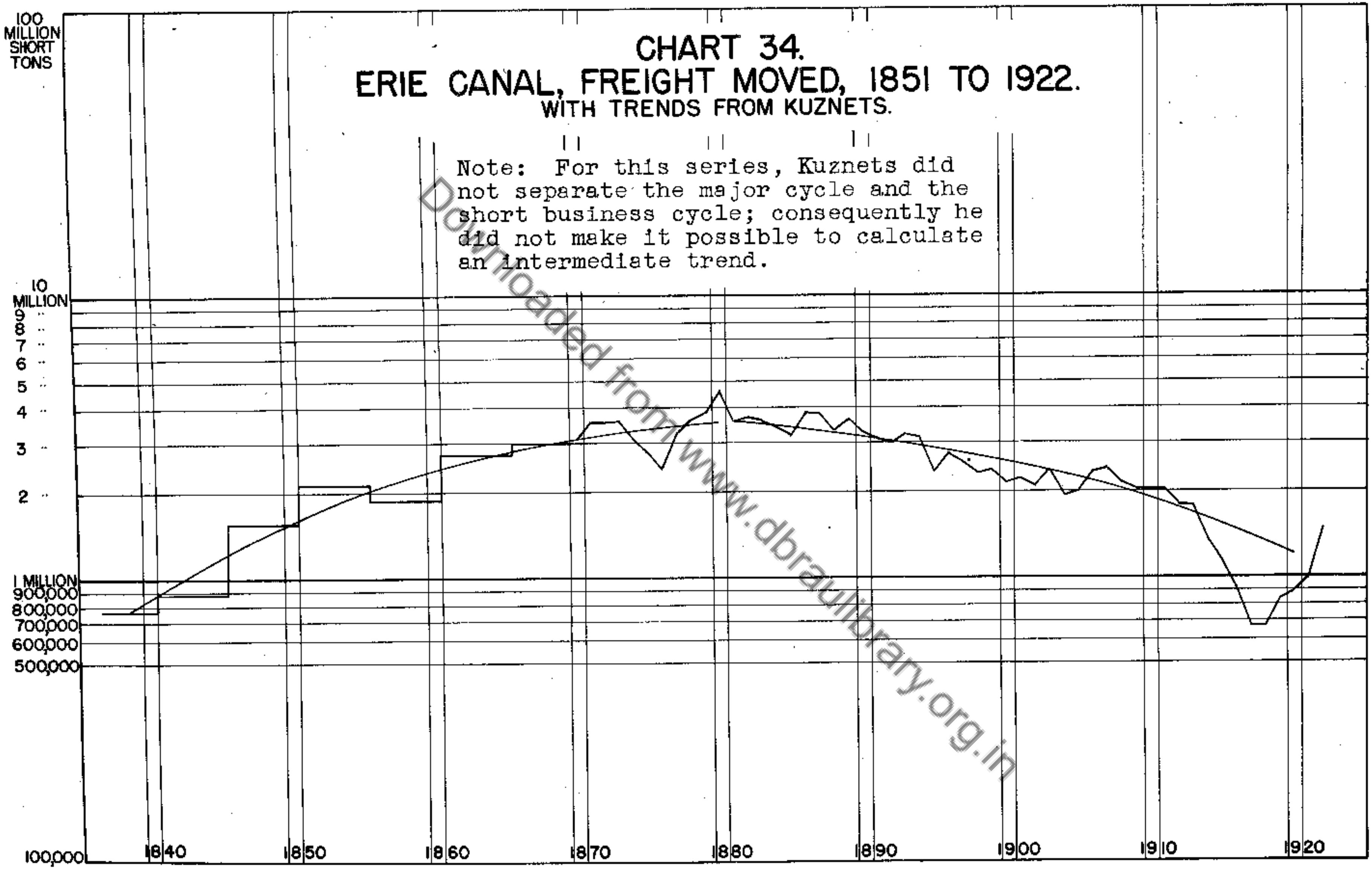
ERIE CANAL, FREIGHT MOVED.

CYCLES BASED ON THE RECORD 1870 TO 1938.



Legend: The short straight lines connect points which show the short business cycle, the ratio of the annual tonnage to SL B.
The curve shows the major cycle, the ratio of SL B to SL M.

CHART 34.
ERIE CANAL, FREIGHT MOVED, 1851 TO 1922.
WITH TRENDS FROM KUZNETS.



Note: For this series, Kuznets did not separate the major cycle and the short business cycle; consequently he did not make it possible to calculate an intermediate trend.

Table U. ERIE CANAL FREIGHT MOVED.

Calculation of standard measures of the two orders of cycles.

Based on the Record 1837 to 1939. Three pages.

1 Year	2a or 4a Freight Moved	23 Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M	31 Percentage Deviation	32 Deviation Squared	1 Year
	(thousands of short tons)									
1870	3083									1870
71	3581									71
72	3563	3600		99	- 1	1				72
73	3603	3420		102	+ 2	4				73
74	3097	3050		101	+ 1	1				74
75	2787	2830		99	- 1	1				75
76	2418	2810		86	-14	196				76
77	3254	3070		106	+ 6	36				77
78	3609	3600	3570	100	0	0	104	+ 4	16	78
79	3820	4030	3610	95	- 5	25	112	+12	144	79
1880	4609	4100	3630	112	+12	144	113	+13	169	1880
81	3599	3830	3640	94	- 6	36	105	+ 5	25	81
82	3694	3600	3630	102	+ 2	4	99	- 1	1	82
83	3587	3500	3620	102	+ 2	4	97	- 3	9	83
84	3390	3430	3600	99	- 1	1	95	- 5	25	84
85	3208	3450	3570	93	- 7	49	97	- 3	9	85
86	3809	3550	3500	107	+ 7	49	101	+ 1	1	86
87	3841	3600	3440	107	+ 7	49	105	+ 5	25	87
88	3322	3530	3350	94	- 6	36	105	+ 5	25	88
89	3674	3420	3270	107	+ 7	49	105	+ 5	25	89
1890	3304	3300	3180	100	0	0	104	+ 4	16	1890
91	3098	3200	3100	97	- 3	9	103	+ 3	9	91
92	2979	3080	3000	97	- 3	9	103	+ 3	9	92
93	3236	2970	2940	109	+ 9	81	101	+ 1	1	93
94	3144	2820	2850	111	+11	121	99	- 1	1	94
95	2356	2690	2770	88	-12	144	97	- 3	9	95
96	2742	2600	2690	105	+ 5	25	97	- 3	9	96
97	2585	2500	2620	103	+ 3	9	95	- 5	25	97
98	2338	2400	2550	97	- 3	9	94	- 6	36	98
99	2419	2320	2480	104	+ 4	16	94	- 6	36	99
1900	2146	2250	2400	95	- 5	25	94	- 6	36	1900

Table U (continued) Erie Canal Freight Moved

(thousands of short tons)

1 Year	2a or 4a Freight Moved	12a Smoothing Line B	24 Smoothing Line M	25 Ratio Actual to SL B %	26 Percentage Deviation	27 Deviation Squared	30 Ratio SL B to SL M	31 Percentage Deviation	32 Deviation Squared	1 Year
1901	2257	2200	2330	103	+ 3	9	94	- 6	36	1901
02	2106	2170	2250	97	- 3	9	97	- 3	9	02
03	2414	2150	2170	112	+12	144	99	- 1	1	03
04	1946	2160	2090	90	-10	100	103	+ 3	9	04
05	2000	2170	2000	92	- 8	64	108	+ 8	64	05
06	2385	2190	1930	109	+ 9	81	113	+13	169	06
07	2416	2200	1840	110	+10	100	119	+19	361	07
08	2177	2170	1780	100	0	0	122	+22	484	08
09	2031	2130	1710	95	- 5	25	124	+24	576	09
1910	2023	2070	1650	98	- 2	4	125	+25	625	1910
11	2032	1970	1580	103	+ 3	9	125	+25	625	11
12	1795	1850	1520	97	- 3	9	122	+22	484	12
13	1788	1670	1480	107	+ 7	49	113	+13	169	13
14	1362	1350	1430	99	- 1	1	94	- 6	36	14
15	1155	1100	1400	105	+ 5	25	79	-21	441	15
16	918	880	1390	104	+ 4	16	63	-37	1369	16
17	675	750	1380	90	-10	100	54	-46	2116	17
18	667	730	1400	91	- 9	81	52	-48	2304	18
19	842	780	1420	108	+ 8	64	55	-45	2025	19
1920	891	910	1450	98	- 2	4	63	-37	1369	1920
21	994	1130	1490	88	-12	144	76	-24	576	21
22	1485	1380	1550	108	+ 8	64	89	-11	121	22
23	1626	1600	1620	102	+ 2	4	99	- 1	1	23
24	1692	1740	1680	97	- 3	9	104	+ 4	16	24
25	1945	1850	1760	105	+ 5	25	105	+ 5	25	25
26	1935	1980		108	+ 8	64				26
27	2048	2160		95	- 5	25				27
28	2536	2350		108	+ 8	64				28
29	2422	2620		92	- 8	64				29
1930	3044	2900		105	+ 5	25				1930
31	3278	3110		105	+ 5	25				31
32	3186	3310		96	- 4	16				32
33	3574	3520		101	+ 1	1				33
34	3645	3700		99	- 1	1				34
35	3898	3880		100	0	0				35
36	4220	3940		107	+ 7	49				36
37	4174									37
38	3349									38
1939	3644									1939

end of the short period for calculation of sd.

end of the short period for calculation of sd.

$\Sigma(d^2) = 2607$

sd $= \sqrt{\frac{2607}{65}} = 6.33\%$, in the short business cycle.
(the full period)

$\Sigma(d^2) = 14672$

sd $= \sqrt{\frac{14672}{48}} = 14.48\%$,
in the major cycle.
(the full period)

(Table U is concluded on the next page.)

Table U (concluded) Eric Canal Freight Moved.

Standard measures of the two orders of cycles.

Time Lengths of the Phases of the Cycles

28
of actual about SL B (the short business cycle)

Cycle Number	Lengths in Years			
	rp	pf	ft	tr
1			.5	.5
2	.5	1.5	1.5	.6
3	.4	1.0	1.0	.3
4	.7	.5	.5	.7
5	.8	1.0	1.3	.7
6	1.0	1.0	.5	.5
7	.5	.8	1.2	1.3
8	1.2	.8	.7	.7
9	.6	1.2	.5	.5
10	.5	.6	.4	.5
11	.5	.5	.5	.3
12	.7	.5	1.0	1.0
13	1.0	1.6	.9	1.5
14	.5	.5	.5	.3
15	.7	.8	.2	.3
16	.7	.8	1.8	1.2
17	.3	.8	1.1	.7
18	.5	1.0	.8	.3
19	.7	.5	1.0	.9
20	.6	.5	.5	.5
21	1.0	1.0	.5	.7
22	.3	.3	.7	1.0
23	1.5	1.0		
Average length	.7 yr.	.8 yr.	.8 yr.	.7 yr.

Total length of typical short business cycle 3.0 yrs.

33
of SL B about SL M (the major cycle)

1	1.8	1.9	2.0	2.3
2	3.3	4.5	6.0	3.3
3	6.8	3.8	4.7	6.3
Average length	4.0 yrs.	3.4 yrs.	4.2 yrs.	4.0 yrs.

Total length of typical major cycle 15.6 yrs.

Percentage Deviations at Peaks and Troughs

29
of actual from SL B (the short business cycle)

At Peak		At Trough	
Year	Deviation	Year	Deviation
1873	2	1876	14
77	6	79	5
80	12	81	6
82	2	85	8
87	9	88	5
89	8	92	3
94	11	95	12
96	5	98	3
99	4	1900	5
1901	3	02	3
03	12	04	10
07	10	09	5
11	3	12	3
13	7	14	1
15	5	17	10
19	8	21	12
22	8	24	3
25	5	27	5
28	8	29	8
30	5	32	4
1933	1	34	1
Average deviation in the short business cycle	6.4% at peak		6.0% at trough

34
of SL B from SL M (the major cycle)

1880	13	1884	4
1888	5	1900	6
1909-10	25		
Average deviation in the major cycle	14.3% at peak		5.0% at trough

SUMMARY OF THE ANALYSES OF THE SEVEN SERIES BY THE TWO METHODS.

KUZNETS' study was not merely an analysis of a group of time series, but a testing of his thesis that the logistic type of curve is almost universally applicable to production and other types of quantity series (not to price series).

THE verdict of the method of smoothing by stages, after testing his thesis, is clearly favorable. Here, with no a priori bias as to the "proper" shape for the trends, a close agreement is found with Kuznets. The few points of difference have been pointed out in the course of Chapter V, and need not be restated.

THERE is no reason that the statistician should not have both arrows in his quiver. Every time series may be analyzed first by the method of smoothing by stages, which will give the unvarnished elements of the series. Then the trend, or the major cycle, the short business cycle, or the seasonal movement may be examined separately, to test any hypothesis, i.e., to see how closely this element of the whole complex movement conforms to a preconceived shape or pattern.